On the Matter of Space: An Already-Unified $\mathfrak{sl}(4,\mathbb{R})$ Invariant Gauge Theory

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(Dated: March 8, 2024)

(This is a work in progress, so forgive blank and duplicate sections.) The $\mathfrak{sl}(4,\mathbb{R})$ algebra as a real-valued, background-dependent representation of the Standard Model is shown to be a basis for unification as a gauge invariance of the metric in General Relativity, a background independent theory. No changes are made to the Einstein field equations, merely a reinterpretation as an identity. The $\mathfrak{sl}(4,\mathbb{R})$ algebra contains both fermionic and bosonic fields as well as scalar, vector and tensor gravitational fields. Mass parameters of those fields are constrained by the volume integral of the the trace of the stress-energy tensor into a non-linear eigenvalue equation giving mass values. This is shown to be true in several cases: static, spherically symmetric gravity, covariant gravitational radiation, EM radiation, black holes, galaxies and the Aharonov-Bohm effect. In different regimes gravity gives normal Schwarzschild gravity, black holes without singularity. The baryon asymmetry and its cosmological implications are discussed. Experiments to validate and refute are proposed.[a]

PACS numbers: 04.20.Cv, 12.10.-g, 95.35.+d, 04.30.-w
Keywords: General relativity formalism, Unified field theories, Cosmology, Dark matter

I. INTRODUCTION

Physics is certainly in a Kuhnian crisis with 95+% of the universe of unknown composition, a status that has persisted for many years. There has also been little progress deconflicting GR and QT, answering the question of unification, or eliminating the problems of singularity and infinities. The last two owe their existence to the lack of an underlying model of matter/particle [1]. The mechanism by which matter both curves, and is guided by spacetime is left unspecified. So is the origin of inertia. It is time for a paradigm shift; time to reassess the underlying assumptions of physical theory. This already-unified gauge hypothesis answers each of the above mentioned issues. It does so while leaving the equations of GR and QT intact and narrowing the ontology rather than widening it. Not only does it provide a great simplification, but a host of problems in theory disappear, and DM and DE appear. It should be expected that some accepted "truths", whose existence is the result of inductive reasoning rather than experiment, will in fact be refuted.

First a pedagogical overview of this theory is given. This is followed by a developmental history showing how each concept is deduced or inferred from the previous. Examples follow.

We certainly know space and time exist, by definition, via rod and clock measurements. If we take Occam's razor seriously, it is incumbent upon physics to determine if physical theory can be based on just that - spacetime. The hypothesis herein is that it is true.

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Since the discovery of the nuclear forces and the success of Quantum Electrodynamics (QED) the focus of unification has primarily been on quantum field theory [2]. Since 3 of the 4 forces are represented in the Standard Model (SM) it seems reasonable to try to quantize the gravitational field to complete the task. However this *could* be a category error. It is possible that quantum principles do not apply to gravity, or that they apply to gravity but not to General Relativity(GR) [3], since it is a background independent theory. Also, unification from a GR perspective is no less valid than through QFT [4]. In either case a connection between the two frameworks should emerge. Although attempts at classical unification have been unsuccessful, there is a good argument to proceed in that direction. Electromagnetism (EM) has one foot in each world. It is a long range force with a successful well developed classical theory like gravitation. It is an integral part of the SM and closely tied to the weak force. Unification of EM with gravity is therefore certain to define the relationship between gravity and quantum theory. Unification is far more than just bringing gravity "into the fold". It requires the solution to two of the most pressing problems in theoretical physics — the relationship between GR and QT, and the description of particle.

The approaches to unification, classical and quantum, all appear to have one aspect in common. They generalize, extend or add degrees of freedom to the mathematical framework. String theory, loop quantum gravity, extra dimensions, non-symmetric connections/torsion, complex metrics, Finsler spaces, etc., have not yet worked. Theories of unification have become increasingly complex and removed from verifiability and falsifiability such as the GUT theories and Supersymmetry. A much simpler formulation is possible. What emerges is a gravitational field as a relativistic gauge field alongside the other forces, and General Relativity as a covariant gauge theory of them all. In this process the Einstein Field Equation (EFE) is promoted to an identity.

II. METHODOLOGY

A subtle yet profound change to the ontological [5] basis of physics can both lead the way to a unified field theory and shed light on the epistemological differences between General Relativity(GR) and Quantum Theory(QT). There is a simple framework for unification that is testable, refutable, and leaves the mathematical structure of general relativity and quantum field theory intact, namely.

$$G_{\mu\nu} + \Lambda\, g_{\mu\nu} = \frac{8\pi\,\kappa}{c^2}\, T_{\mu\nu}, \tag{1}$$

for the Einstein Field Equation (EFE) and

$$F^{\mu\nu}{}_{;\nu} = j^{\mu}, \quad F_{[\mu\nu,\lambda\,]} = 0, \tag{2}$$

$$\nabla_{\nu} F^{\mu\nu} = j^{\mu}, \quad F_{[\mu\nu,\lambda\,]} = 0, \tag{3}$$

for the Maxwell Equations (ME). (The sign convention being used is Mostly Minus (MM) for the metric).

This already-unified gauge field theory is an extremely simple theory. It eliminates singularities from the theory. It reconciles the background-independent and background-dependent foundations of general relativity and quantum theory, respectively. It eliminates the need to "put in by hand" the right hand side of the EFE. It can also provide the missing picture [6] for quantum theory. It solves several outstanding problems in physics and naturally includes Dark Matter and Dark Energy. The following set of deductions and inferences provides a compelling argument for this already-unified gauge field theory. The reasoning is based upon two principles that are not entirely independent. They are actually inferred from the theory, not the other way around.

Regardless of whether spacetime is discrete or continuous at the smallest scales, it will be treated here as a continuum, possibly limiting the scale of applicability.

A. Axioms

1. The only necessary field is the field of event displacements.

   Historically the field concept was introduced to avoid action-at-a-distance [7]. A dynamical spacetime accomplishes this for gravity in GR through the metric tensor; matter curves spacetime, spacetime guides matter. There is no such view for the EM forces. In that case the space is mitigated by a field. It is necessary to have a dynamical spacetime formulation of EM to fit it into GR. The use of the word mitigated above is literal: to lessen. Assuming matter at a distance cannot interact without fields yields a contrapositive: if material particles do interact without fields, then they must not be "at a distance". There is only one way this can happen: matter itself must extend through spacetime, as part of the same continuum, so that one particle can smoothly meld into another. So in order to eliminate the field concept, a particle must be made up of the same "stuff" as spacetime with most of it fairly well localized to appear as a particle. Two particles can therefore interact using a field as an intermediary, or a dynamical spacetime. The field concept becomes superfluous if it can be shown that the event displacement field solves the same equations. Furthermore, asserting that "Newtonian space and time and the gravitational field are the same entity" [8], particles must be made of gravitational fields as well. This is shown below and is the implementation of what has been called the *Maxwellian dream* [1]. Empty space is not empty. It has mass. It *is* mass. What we see as quantum fluctuations are tiny fluctuations in this "stuff" due to the motions of everything. What we see as DM is over-densities of this "stuff". Further more, there must therefore be a gravitational temperature associated with these fluctuations. What we see as DE is the result of stretching it. This also explains why ΛCDM works as well as it does despite the clumpiness of the cosmic web.

2. The field should be free from singularities.

   The presence of singularities in GR is unacceptable to many general relativists [9]. This was also Einstein's viewpoint [10]; an acceptable theory has to work everywhere. Infinities are unmeasurable. Particles must be represented by a finite matter field in some finite region. The observables of spacetime are distance and duration. These are specified by the metric tensor. Therefore, for any real configuration of matter there must exist a coordinate system that results in measurable intervals everywhere. As will be shown, the removal of singularities is an automatic byproduct of the gauge theory.

B. The Gauge Field

GR is a gauge theory. The gauge invariance of GR is the invariance of the spacetime interval under coordinate transformations (diffeomorphisms). In contrast, the gauge fields of the forces are based upon the (not necessarily diffeomorphic) metric gauge invariance of the 4-volume: in particular the Lie group $SL(4,\mathbb{R})$. Coordinate gauge invariance of the 4-volume element τ, for example, is

$$\sqrt{-\bar{g}}\,d\bar{\tau} = \sqrt{-g}\,d\tau, \tag{4}$$

while its metric gauge invariance is

$$\sqrt{-\bar{g}}d\tau = \sqrt{-g}d\tau. \tag{5}$$

It has been shown that all the Lie groups of the SM can be found within the corresponding algebra of $\mathfrak{sl}(4,\mathbb{R})$ and its inner and outer automorphisms [11].

The Maxwell equations, Eqs. (2) are analogous to the equations of fluid flow, complete with sources, sinks and vortices. This was noted by Maxwell early on and there were attempts to mechanize the field with a quasi-elastic ether model [12]. Riemann attempted to unify gravity and electromagnetism with such a model [13]. The approach was to assume space contained some kind of substance that could flow or spin. These ideas were unworkable. Michelson and Morely showed that there is no lumeniferous ether [14]; such a substance would permeate space and serve as a dynamical medium for EM kinematics. Therefore the analogy is either an accidental coincidence, or it represents some other kind of motion. There is only one other possibility for such a displacement field. *It is that the Maxwell equations represent a transformation of spacetime points themselves, rather than some substance occupying spacetime points.* Just accepting it as a "field" admits that the structure of the equations is a coincidence. Such an acceptance also introduces a new elementary object that requires its relationship to gravity be separately defined, unnecessarily complicating the ontology [15] – Occam's razor.

The transformation of events can be described mathematically in the same way as that of a deformable physical medium. Consider an infinitesimal displacement, $\boldsymbol{\xi}$, in the neighborhood of a small volume element in a 3 dimensional Euclidean space. It is composed of a rotation, a compression (extension or shear), and a translation [16].

$$\begin{aligned}\xi^i\left(x^j\right) &= \xi^i(0) + \xi^i{}_{,j}dx^j + \mathcal{O}\left[dx^2\right] \\ &\approx \xi^i{}_0 + g^{\mathrm{il}}\left[\frac{1}{2}\left(\xi_{l,j}+\xi_{j,l}\right)dx^j + \frac{1}{2}\left(\xi_{l,j}-\xi_{j,l}\right)dx^j\right], \\ &\qquad \{i,j,l\}\in\{1,2,3\}.\end{aligned} \tag{6}$$

This can be generalized to a 4-dimensional pseudo-Euclidian base space, having metric g, that is tangent to the Riemannian manifold at some point. In that case temporal displacements as well as spatial displacements are both taken to be functions of space and time.

$$\begin{aligned}\xi^\mu\left(x^0,x^j\right) &\approx \xi^\mu(0,0) + \xi^i{}_{,0}dx^0 + \xi^0{}_{,j}dx^j \\ &+ g^{\mathrm{il}}\left[\frac{1}{2}\left(\xi_{l,j}+\xi_{j,l}\right)dx^j + \frac{1}{2}\left(\xi_{l,j}-\xi_{j,l}\right)dx^j\right], \\ &\qquad \{i,j,l\}\in\{1,2,3\},\ \mu\in\{0,1,2,3\}.\end{aligned} \tag{7}$$

For infinitesimal displacements this becomes

$$\begin{aligned}d\xi^\mu\left(x^\nu\right) &= g^{\mu\lambda}\left[\frac{1}{2}\left(\xi_{\lambda,\nu}+\xi_{\nu,\lambda}\right)dx^\nu\right. \\ &\left.+\frac{1}{2}\left(\xi_{\lambda,\nu}-\xi_{\nu,\lambda}\right)dx^\nu\right],\ \{\mu,\nu,\lambda\}\in\{0,1,2,3\}.\end{aligned} \tag{8}$$

Putting this into covariant form allows for arbitrary coordinate systems in the base space.

$$\begin{aligned}\xi^\mu{}_{;\nu}dx^\nu &= g^{\mu\lambda}\frac{1}{2}\left(\xi_{\lambda;\nu}+\xi_{\nu;\lambda}\right)dx^\nu \\ &+ g^{\mu\lambda}\frac{1}{2}\left(\xi_{\lambda;\nu}-\xi_{\lambda;\mu}\right)dx^\nu,\ \{\mu,\nu,\lambda\}\in\{0,1,2,3\}.\end{aligned} \tag{9}$$

Expressing this in terms of the symmetry properties of the displacement field,

$$\begin{aligned}\xi^\mu{}_{;\nu}dx^\nu &= \frac{1}{2}g^{\mu\lambda}\left(\xi_{\lambda;\nu}+\xi_{\nu;\lambda}\right)dx^\nu \\ &+ \frac{1}{2}g^{\mu\lambda}\left(\xi_{\lambda;\nu}-\xi_{\lambda;\mu}\right)dx^\nu \\ &= g^{\mu\lambda}\sigma_{\lambda\nu}dx^\nu + g^{\mu\lambda}\alpha_{\lambda\nu}dx^\nu\end{aligned} \tag{10}$$

with σ the symmetric tensor and α antisymmetric. The relationship between the displacement and the metric tensor, g, then follows. Assume a displacement field is introduced into a locally flat region of space with coordinates x^μ and metric $\bar{g}$,

$$\bar{g} = \begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix},\ x^\mu = \left(x^0,x,y,z\right) \tag{11}$$

so that

$$ds^2 = \bar{g}_{\mu\nu}dx^\mu dx^\nu. \tag{12}$$

An infinitesimal displacement of the points would cause two events with the coordinate separation dx^μ to have a new coordinate separation,

$$d\bar{x}^\mu = \left(\delta^\mu_\nu + \epsilon\xi^\mu{}_{;\nu}\right)dx^\nu,\ \epsilon = \frac{1}{N} << 1. \tag{13}$$

A finite displacement field gives

$$\begin{aligned}d\bar{x}^\mu &= \underset{N\to\infty}{\mathrm{Limit}}\left[\left(\delta^\mu_\nu + \frac{1}{N}\xi^\mu{}_{;\nu}\right)^N\right]dx^\nu \\ &= \left(\underset{N\to\infty}{\mathrm{Limit}}\left[\left(\mathbf{I}+\frac{\boldsymbol{\zeta}}{N}\right)^N\right]\right)^\mu{}_\nu dx^\nu \\ &= \left(e^{\boldsymbol{\zeta}}\right)^\mu{}_\nu dx^\nu,\end{aligned} \tag{14}$$

where $\left(e^{\boldsymbol{\zeta}}\right)^\mu{}_\nu$ is the (μ,ν) component of the tensor obtained by exponentiating the tensor whose components are $\zeta^\mu{}_\nu = \xi^\mu{}_{;\nu}$ and $\mathbf{I}$ is the identity tensor.

The displacement can be considered either as new coordinates for the points using the old metric, $\bar{g}$, or as a new, transformed metric, g, using the old coordinates.

$$\begin{aligned}ds^2 &= \bar{g}_{\mu\nu}dx^\mu dx^\nu \longrightarrow \bar{g}_{\mu\nu}d\bar{x}^\mu d\bar{x}^\nu \\ &= \bar{g}_{\mu\nu}\left(\left(e^{\boldsymbol{\zeta}}\right)^\mu{}_\lambda dx^\lambda\right)\left(\left(e^{\boldsymbol{\zeta}}\right)^\nu{}_\tau dx^\tau\right) \\ &= \left(\left(e^{\boldsymbol{\zeta}}\right)^\mu{}_\lambda \bar{g}_{\mu\nu}\left(e^{\boldsymbol{\zeta}}\right)^\nu{}_\tau\right)dx^\lambda dx^\tau \\ &= g_{\lambda\tau}dx^\lambda dx^\tau\end{aligned} \tag{15}$$

This defines the relationship between spacetime displacements and the metric tensor;

$$\boxed{g_{\lambda\tau} = \left(e^{\boldsymbol{\zeta}}\right)^{\mu}_{\lambda} \bar{g}_{\mu\nu} \left(e^{\boldsymbol{\zeta}}\right)^{\nu}_{\tau}}. \tag{16}$$

Since these displacement fields, $\boldsymbol{\zeta}$, are linear operators, and they satisfy the necessary $\mathfrak{sl}(4,\mathbb{R})$ commutation relationships, they are also quantum fields. A set of bases are provided in terms of 4x4 real matrices, bi-quaternions and Dirac gamma matrices [11].

1. Background Dependent vs Background Independent

Thus the transformed metric using the old coordinates can be thought of as the consequence of using the "wrong" coordinates, that consequence giving rise to "fictitious" or inertial force fields, the usual view of gravity. This gauge transformation of the metric tensor is a type of factorization of the metric like using tetrads, but different in meaning, and based on general tensors. The transformations among the -variant forms of $\boldsymbol{\zeta}$ and its covariant derivatives still use the base space metric since they are measured locally, and with respect to the old coordinates.

All the formulations of physical laws using the field $\boldsymbol{\zeta}$ and its base space metric are therefore "background dependent"; they rely on background structures [5] which may vary from event to event. Their phenomenologies are derived in that local space, depend on its metric, and as such cannot be expected to have a generally covariant form. *They are flat-space laws.* However, Eq. (16) defines their relationship to the exact metric tensor and therefore defines their participation in a generally covariant "background independent" theory, GR. That theory now involves all the forces, at all scales, classical as well as quantum fields.

The flat space laws are subject to quantization which relies on their background dependency. These quantum fields can be mapped into $\boldsymbol{\zeta}$ based upon their analytic and geometric properties, when expressed as functions of the coordinates [11]. This gives them a classical picture [17]. Although the terms "field" and "displacement field" are being used, no new "fields" are being introduced; these mathematical fields just describe the displacement of events from their base-space coordinate locations in spacetime. Now the connections among the Maxwell equations, spacetime flows and rotations, and the metric tensor can be specified.

C. The Electromagnetic Field

The antisymmetric part of the displacement field in Eq. (10) represents rotations and flows of events with respect to the base space coordinate system. It is also an exact tensor so automatically satisfies 2 of the Maxwell equations. It is therefore taken to be (proportional to) the electromagnetic field.

$$\begin{aligned}\alpha_{\mu\nu} = \frac{1}{2}\left(\xi_{\mu,\nu} - \xi_{\nu,\mu}\right) = \frac{1}{2}\xi_{\ [\mu,\nu\ ]} \\ \equiv f_{\mu\nu} \propto F_{\mu\nu} = \eta f_{\mu\nu},\end{aligned} \tag{17}$$

with the Maxwell equations as in Eq. (2) and the Cartesian components of the microscopic electromagnetic field tensor and 4-vector potential represented as

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{pmatrix}, \tag{18}$$

$$F_{\mu\nu} = \phi_{\mu,\nu} - \phi_{\nu,\mu} \qquad \phi^{\mu} = \left(A^0, A^{\iota}\right), \tag{19}$$

in Heaviside-Lorentz units. There is a problem with interpreting the electromagnetic field as having flows. The source equations seem to require sources (sinks) out of (into) which whatever is flowing is appearing (disappearing) [12]. This can be explained by imbuing spacetime with a non-simply connected topology [18, 19]. Mathematically that introduces a significant complexity. However it is unnecessary. Consider the electric field component of $f_{\mu\nu}$.

$$f_{0i} = \frac{1}{2}\left(\xi_{0,i} - \xi_{i,0}\right) = -\boldsymbol{e} \propto -\boldsymbol{E} = \nabla A_0 + \frac{\partial \boldsymbol{A}}{\partial t} \tag{20}$$

This "flow" is a space-time rotation. It has two terms. The time derivative represents a flow of spatial points that has to go somewhere for any steady state. However for the electrostatic case, with no other matter or energy present, the vector potential, $\mathbf{A}$, or its divergence, is chosen to be zero (Coulomb gauge). In the electrodynamic case the radiation has an oscillatory (or transient in the case of non-periodic induction) vector potential. In neither case is a novel topology required. For an electrostatic field,

$$\boldsymbol{E} = -\nabla A_0 \propto -\xi_{0,i}, \tag{21}$$

sources (electric charges) cause a gradient of the time displacement, A_0. It appears that a clock placed near an electric charge and then moved will be offset ahead or behind depending upon the sign of the charge. This may be true on a particular trajectory as determined by the connections, which may have factors linear in E, but are coordinate dependent. The metric ultimately determines the "rate" in any case. The size of any such offset or rate effect is determined by the proportionality constant, η, between $\boldsymbol{e}$ and $\boldsymbol{E}$. Magnetic fields correspond to purely spatial rotations.

This completely geometrized electromagnetic field is a gauge transformation on events under $\mathfrak{sl}(4,\mathbb{R})$. Consider the particular case where the symmetric tensor in Eq. (10) is zero. In that particular gauge the antisymmetric electromagnetic field tensor is like a Lorentz transformation, with two important differences:

1. It is a physical transformation of events (active transformation), not coordinates (passive transformation) and

2. It is local not global. It varies from event to event. That is, it is a second order gauge transformation.

That explains why EM theory is Lorentz-invariant; mathematically it is a local Lorentz transformation. However, it is more than just that; the vector potential is primary in real physical configurations of matter where symmetric components may be present. In particular, if

$$\sigma_{\lambda\nu} = \frac{1}{2}\left(\xi_{\lambda;\nu} + \xi_{\nu;\lambda}\right) = 0 \tag{22}$$

then ξ^μ is a Killing vector field so the metric possesses a hidden symmetry. The symmetry the Killing field admits is the electromagnetic field.

$$\begin{aligned} g_{\mu\nu} = \bar{g}_{\mu\nu} \Rightarrow \alpha_{\mu\nu} &= \frac{1}{2}\left(\xi_{\mu,\nu} - \xi_{\nu,\mu}\right) \\ &\equiv f_{\mu\nu} \propto F_{\mu\nu} \neq 0. \end{aligned} \tag{23}$$

In this particular gauge, since $\sigma_{\mu\nu} = 0$, and $\alpha_{\mu\nu}$ is antisymmetric,

$$g_{\mu\nu} = \left(e^{\boldsymbol{\alpha}}\right)^\mu_\lambda \bar{g}_{\mu\nu} \left(e^{\boldsymbol{\alpha}}\right)^\nu_\tau = \bar{g}_{\lambda\tau}, \tag{24}$$

so that the electromagnetic field is therefore another gauge symmetry, one that is a diffeomorphism, but one which leaves the metric itself unchanged. This means no curvature and the electromagnetic field in this gauge is not a source of gravity. This is not true in other gauges like other EM fields or EM fields combined with gravity. This will be seen below in the case of the The Aharonov-Bohm effect.

D. The Already-Unified Gauge Field Hypothesis

Consider the generalization of Eq. (16) where the gauge field ζ is any second rank tensor that is a linear combination of the generators of $\mathfrak{sl}(4,\mathbb{R})$. ζ can be decomposed into a tensor with zero divergence and one with zero curl (anti-symmetrized derivative).

$$\zeta_{\mu\nu} = \xi_{\mu\nu} + f_{\mu\nu}, \quad \xi^{\mu\nu}_{\ \ ;\nu} = 0, \quad f_{\ [\mu\nu,\lambda]} = 0. \tag{25}$$

This means f is closed and therefore exact, admitting a vector potential

$$f_{\mu\nu} = \bar{\phi}_{\mu,\nu} - \bar{\phi}_{\nu,\mu} \tag{26}$$

and therefore

$$f^{\mu\nu}_{\ \ ;\nu} = j^\mu \neq 0, \quad \sigma_{\ [\mu\nu,\lambda]} \neq 0, \tag{27}$$

in general. In addition, the vector field $\bar{\phi}$ can be decomposed into one with zero divergence and one with zero curl.

$$\begin{aligned} \bar{\phi}^\mu = \phi^\mu + \bar{g}^{\mu\nu}\eta_\nu, \quad \phi^\mu_{\ ;\mu} = 0, \\ \eta_{\ [\mu,\nu]} = 0 \Rightarrow \eta_\mu = \lambda_{,\mu}. \end{aligned} \tag{28}$$

So any general tensor field $\boldsymbol{\zeta}_{\mu\nu}$ can be derived from a vector field, ϕ, that satisfies all the Maxwell equations Eq. (2) and the Lorentz condition,

$$\begin{aligned} f_{\mu\nu} = \frac{1}{2}\left(\xi_{\mu,\nu} - \xi_{\nu,\mu}\right) &= \bar{\phi}_{\mu,\nu} - \bar{\phi}_{\nu,\mu} \\ &= \phi_{\mu,\nu} - \phi_{\nu,\mu}, \quad \phi^\mu_{\ ;\mu} = 0, \end{aligned} \tag{29}$$

plus a tensor field, ξ, derived from tensor, vector and scalar elements;

$$\xi_{\mu\nu} = \chi_{\mu\nu} + \left(\phi_{\mu;\nu} + \phi_{\nu;\mu}\right) + \lambda_{,\mu;\nu}, \tag{30}$$

where the tracelessness ensures the Lorentz gauge:

$$\chi^{\mu\nu}_{\ \ ;\nu} = 0, \quad \chi^\nu_{\ \nu} = 0, \quad \bar{g}^{\mu\nu}\lambda_{,\mu;\nu} = \Box\lambda = 0 \tag{31}$$

It is also true that any second rank tensor can be decomposed into an antisymmetric tensor, a traceless symmetric tensor and a multiple of the metric tensor. These facts will be used to identify the physical fields that these gauge fields represent. As will be shown, in each example case below, the gauge field is traceless as appropriate for an $\mathfrak{sl}$ algebra.

It is the main hypothesis here that these displacement fields, ζ,

$$\boxed{\zeta_{\mu\nu} = \chi_{\mu\nu} + \left(\phi_{\mu;\nu} + \phi_{\nu;\mu}\right) + \lambda_{,\mu;\nu} + f_{\mu\nu}}, \tag{32}$$

are what appear in the base space as force fields and matter. More generally all linear combinations of the generators of the $\mathfrak{sl}(4,\mathbb{R})$ algebra are possible, including real spinor representations of the group [11]. The symmetric fields generate gravity; scalar, vector and tensor aspects. The antisymmetric vector field is EM as part of the electroweak field. For a given configuration of matter the equations of these fields are known: Klein-Gordon, Maxwell, Dirac, Proca, etc. The gauge field fluctuations must give rise to a gravitational temperature out of which virtual particles are continuously created and destroyed.

This is the implementation of ***A***.1. Thus EM is incorporated into GR simply by identifying it as that part of the displacement gauge field solving the Maxwell equations. EM remains unchanged, as a flat space theory, and so does the formalism of GR, as a covariant theory. The way EM enters into GR however is very different. The consequences of this will be dealt with below.

The physical significance of this invariance under the group $SL(4,\mathbb{R})$ is that the aether seems like an elastic material. Spacetime can bend and stretch but not tear. The accumulation at one event must be compensated at another event. This crucial fact is at the heart of both DM and DE. The fact that there is more physics hidden within the metric than simply gravity is fundamental to this theory. That is how it is "already-unified". All the forces, including gravity, are slipped in underneath the theory on the left hand side of the EFE. This is instead of putting all the forces *except* gravity on the right hand side. The matter tensor is the consequence of this procedure rather than a constraint upon the metric. No changes are made to the EFE except that it is now the EFI - the Einstein Field Identity. This explains how the geometry of space determines its matter content.

E. The Gravitational Field

1. *The Gauge Field of (Local) Single-Component Tensor Gravity*

The symmetric part of the displacement in Eq. (10) represents compressions (extensions or shears) of space-time points. It is not an exact tensor so its components involve the connections. This means the tensor can be transformed away (locally) by a judicious choice of coordinates. This is a key property of the gravitational field. Also gravity is often depicted as stretches or compressions of the "fabric" of space and time. Identifying the symmetric field with gravity is therefore consistent with the prevalent picture. The exact connection between the symmetric field, $\sigma_{\mu\nu}$, and gravity can be deduced from the Schwarzschild solution to the EFE with Λ=0,

$$G_{\mu\nu} = \frac{8\pi\kappa}{c^2} T_{\mu\nu}, \tag{33}$$

The spherically symmetric, static, free space solution of Eq. (33),

$$G_{\mu\nu} = 0, \tag{34}$$

in spherical coordinates $\left(x^0, r, \theta, \phi\right)$, in the region exterior to some mass, M, is the metric

$$g_s = \begin{pmatrix} 1-\frac{2m}{r} & 0 & 0 & 0 \\ 0 & -(1-\frac{2m}{r})^{-1} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin^2(\theta) \end{pmatrix} \tag{35}$$

with

$$m = \frac{\kappa M}{c^2}. \tag{36}$$

The EFE relates the metric tensor to matter content. However, the general-relativistic matter density is different from the classical value for a given system. This means the metric tensor must be known in order to calculate the correct relativistic value. While it is true that the Einstein tensor can be set to 0 and the equation solved, this may not correspond to a physically realizable configuration, at least not the one intended.

It is well established that there is something appearing to be Dark Matter (DM) everywhere. Its signature is found in the Cosmic Microwave Background (CMB), galaxy rotations curves, and both stellar and galactic clusters. There is strong evidence for Dark Energy (DE) everywhere as well. Space is also filled with the residuals of all the matter in it as shown below. From a quantum perspective, regardless of the interpretation of the Casimir effect [20], "empty" space appears to be filled with fluctuating zero-point fields. It is not known what the impact of the vacuum fluctuations are on the metric tensor. This is one of the major outstanding puzzles in physics with a 120 order of magnitude discrepancy between theory and observation. In any case $G = 0$ is wrong for so-called empty space. Setting it equal to zero may seem like a benign approximation, but it is what produces singular solutions in both GR and QFT (point particles) [1]. These issues are strong evidence of a fundamental lack of understanding of the nature of space. It is also the cause of having to resort to unnecessary "fields" to obviate action at a distance.

If there is a methodology for determining a gauge field instead of a pre-specified stress-energy tensor, $T_{\mu\nu}$, these problems may be solved. The requirements are that the method should be covariant, treat all forces the same, pass the same experimental tests as the Schwarzschild solution and correspond to the classical equation in the low-velocity and weak-field limit [21]. It is easy to infer the appropriate gauge field for a spherically symmetric mass.

The $(0,0)$ and $(1,1)$ components of the Schwarzschild metric, g_s, look like the first two terms in a series expansion squared,

$$1-\frac{2m}{r} \approx (1+\epsilon)^2, \quad (1-\frac{2m}{r})^{-1} \approx (1+\epsilon)^{-2} \tag{37}$$

with $\epsilon = -m/r$. Comparing with Eq. (14) this can be seen to be an approximation of

$$\boldsymbol{\zeta}^{\mu}_{\nu} = \begin{pmatrix} \Phi & 0 & 0 & 0 \\ 0 & -\Phi & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \boldsymbol{\zeta}_{\mu\nu} = \begin{pmatrix} \Phi & 0 & 0 & 0 \\ 0 & \Phi & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{38}$$

with $\zeta_{\mu\nu}$ symmetric and

$$\Phi = \Phi(r) = -\frac{m}{r}. \tag{39}$$

So Eq. (16) gives the new metric for this gauge transformation. For a static, spherically symmetric mass it is given by

$$\begin{aligned} g &= \left(e^{\boldsymbol{\zeta}}\right)^{\mu}_{\lambda} \bar{g}_{\mu\nu} \left(e^{\boldsymbol{\zeta}}\right)^{\nu}_{\tau} \\ &= \begin{pmatrix} e^{2\Phi} & 0 & 0 & 0 \\ 0 & -e^{-2\Phi} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\mathrm{Sin}^2(\theta) \end{pmatrix}. \end{aligned} \tag{40}$$

So in this case the symmetric gauge field for gravitation, $\boldsymbol{\zeta}_{\mu\nu}$, is simply the classical gravitational potential energy per unit mass as measured in the base frame. The mixed tensor, $\boldsymbol{\zeta}^{\mu}_{\nu}$, is simply a scalar field times a scalar generator of $\mathfrak{sl}(4,\mathbb{R})$ [11], a single component tensor field. Now that the metric of Eq. (40) is determined, Φ will be generalized to a gauge potential valid everywhere, not limited to the exterior region of some mass.

This is not the only type of gravitational field – any symmetric (or mixed) gauge, scalar, vector or tensor, results in a tensor, regardless of how it appears locally.

2. The Matter Tensor

Putting Eq. (40) into the EFE, Eq. (33), yields

$$T^{\mu}_{\ \nu} = \frac{c^2}{8\pi\kappa} \times \begin{pmatrix} \Phi_1(r) & 0 & 0 & 0 \\ 0 & \Phi_1(r) & 0 & 0 \\ 0 & 0 & \Phi_2(r) & 0 \\ 0 & 0 & 0 & \Phi_2(r) \end{pmatrix}, \tag{41}$$

where

$$\Phi_1(r) = -\frac{-1 + e^{2\Phi(r)} + 2e^{2\Phi(r)} r\Phi'(r)}{r^2},$$
$$\Phi_2(r) = -\frac{e^{2\Phi(r)}\left(2\Phi'(r) + 2r\Phi'(r)^2 + r\Phi''(r)\right)}{r}. \tag{42}$$

3. The Laue Scalar

The trace of the matter tensor will be referred to as the Laue scalar following Einstein [22]. It plays a central role here as the invariant density. Its 4-volume integral, invariant under $\mathfrak{sl}(4,\mathbb{R})$ metric gauge transformation, gives the rest mass of the particle formed by the field's self-interaction as shown below. The matter tensor of Eq. (41) has some properties that can be deduced at once from its form. The radial pressure is different from the other 2 orthogonal stresses; this cannot be thought of as a Newtonian fluid. Thus the TOV equation does not apply. The radial pressure is negative (when the energy density is positive), consistent with an attractive force. The ratio of the radial pressure to the energy density is -1 — the hallmark of Dark Energy (DE). However, as will be shown, the sum of all the stresses, and its gradient, can be quite different.

4. The Mass

The invariant matter density given by the Laue scalar is

$$\rho(r) = T^{\alpha}_{\ \alpha} = \frac{c^2\left(\Phi_1(r) + \Phi_2(r)\right)}{4\pi\kappa} \tag{43}$$

Integrating the scalar density, $\rho(r)\sqrt{-g}$, over a 4-volume, τ, gives a scalar.

$$I(r) = \int_{\tau} \rho(r')\, 4\pi r'^2 dr' dt = 0, \tag{44}$$

since the integrand is static. So integrating the scalar density, $\rho(r)\sqrt{-g}$, over a spherical volume with radius,

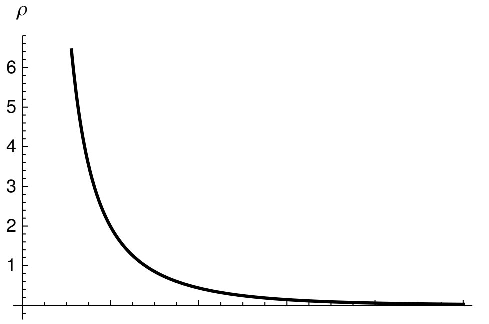

FIG. 1. The invariant matter density (Laue scalar) for ($\kappa = c = m = 1$).

r, gives an invariant, the rest mass, contained within the volume.

$$M(r) = \int \rho(r')\, 4\pi r'^2 dr'$$
$$= \frac{rc^2\left(1 - e^{2\Phi(r)} - e^{2\Phi(r)} r\Phi'(r)\right)}{\kappa} \tag{45}$$

It should be noted that since $T \neq 0$ there may be non-isotropic pressure and pressure gradients causing non-geodesic motion.

5. The Exterior Solution

Eq. (39) is the solution of Poisson's equation,

$$\nabla^2\Phi = 4\pi\rho, \tag{46}$$

in the region outside the matter distribution where $\rho = 0$. This is a local interpretation. It is a Newtonian potential valid everywhere except the origin, where there is a singularity to prevent the empty space solution. From the perspective of the EFE, Fig. 1 shows the matter as continuous distribution of energy in covariant form.

Eq. (39) however is still just a better approximation; the gauge field is still just an exterior solution. The coordinate singularity at r=2m (the event horizon) is eliminated but the essential singularity at $r = 0$ still exists. For $r >> 2m$ this is the same metric as the Schwarzschild solution, but now $G \neq 0$ except in the limit $r \to \infty$.

For Φ(r) as in Eq. (39), Eq. (43) gives the invariant density which is shown in Fig. 1 for $\kappa = c = m = 1$.

For Φ(r) as in Eq. (39), Eq. (45) gives the enclosed mass at a radius, r,

$$M(r) = \frac{\left(1 - e^{-\frac{2m}{r}}\left(1 + \frac{m}{r}\right)\right) rc^2}{\kappa}. \tag{47}$$

The limit as r→ ∞ is

$$\lim_{r\to\infty}\left(M(r)\right) = \frac{mc^2}{\kappa} = M. \tag{48}$$

Fig. 2 shows that the mass increases smoothly from 0

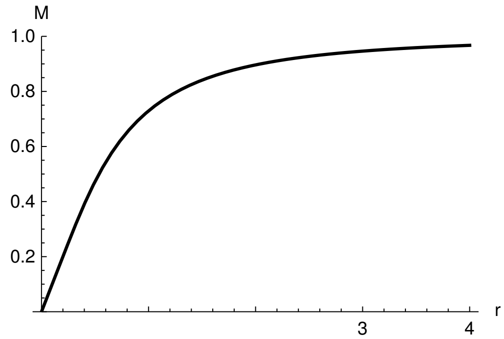

FIG. 2. The mass within sphere of radius, r ($\kappa = c = m = 1$).

toward its limit value quickly. The matter distribution given by Eq. (45) completely accounts for the entire mass, M. This is only valid if at each point, r, the mass appears contained in some sphere $r_s \leq$ r according to the base frame. If the whole of Fig. 2 is taken, then this is an exterior solution of the local description of a point particle. There still is the singularity at the origin to remove in order to satisfy $\boldsymbol{A}$.2. So along with Eq. (46) the gauge field is neither relativistic nor complete.

6. *The Free Relativistic Interior Solution*

The mass now has to be modeled is this simplified ontology. There is only one choice. Locally, this is a single component, i.e., scalar field, Φ, as in Eq. (38) (A tensor field with one independent component). According to relativity, its equation of state is the Klein-Gordon (K-G) equation and the solutions are bosons, which are superposable. Consider the K-G equation for a spherically symmetric field.

$$\Box\Phi + k^2\Phi = 0 \tag{49}$$

The static solution is

$$\Phi(r) = \pm m\frac{e^{-kr}}{r}, \tag{50}$$

and the corresponding dynamic solution for

$$\Box\Phi + \left(\omega^2 + k^2\right)\Phi = 0 \tag{51}$$

is

$$\Phi(t,r) = m\mathrm{Cos}(\omega t + \alpha)\frac{e^{-kr}}{r}. \tag{52}$$

Any potential that avoids the singularity provides a picture of matter as a self-contained region of gravitational energy in a state bound by the mass's own gravitational attraction. The K-G field assumes only the relativistic relation between mass and energy, and is therefore "generic". Any field, regardless of its equation of state, has to satisfy the K-G equation on a component-by-component basis in addition to any other state specific conditions that are present. This is true regardless of scale from elementary particles to galactic clusters.

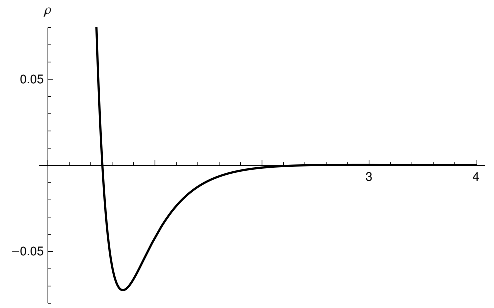

FIG. 3. Laue scalar for a Yukawa gauge field ($\kappa = c = m = 1$).

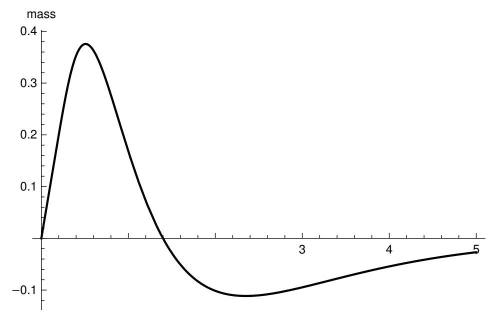

FIG. 4. Mass profile for a Yukawa gauge field ($\kappa = c = m = 1$).

Putting Eq. (50) into Eq. (43) and choosing the minus sign gives the profile for the Laue scalar as shown in Fig. 3.

Putting the same potential into Eq. (45) gives,

$$\begin{aligned} M(r) = \frac{rc^2}{\kappa}\left(mr\left(\frac{e^{-kr}}{r^2} + \frac{ke^{-kr}}{r}\right)\right. \\ \left.\times\left(-e^{-\frac{2me^{-kr}}{r}}\right) - e^{-\frac{2me^{-kr}}{r}} + 1\right) \end{aligned} \tag{53}$$

the mass contained within a radius, r as shown in Fig. 4.

The limit as r$\to\infty$ is

$$\lim_{r\to\infty}\left(M(r)\right) = 0. \tag{54}$$

Exactly half the mass is positive energy and half negative energy. This is a remarkable property of the Yukawa potential in GR. Since this is the static solution, is it also a zero-energy solution. The binding energy is equal and opposite to the mass-energy. For a dynamic solution, which needs a time-dependent metric, the mass profile oscillates like a standing spherical wave, but maintains equal and opposite mass-energies at each instant.

From a quantum perspective Eq. (49) gives

$$\Box\Phi + k^2\Phi = -\nabla^2\Phi + k^2\Phi = 0, \tag{55a}$$

$$\Rightarrow \hat{p}^2\Phi = -k^2\Phi, \tag{55b}$$

so the field momentum and mass are imaginary since k is real - a tachyonic field. The reason for this is that it is a momentum operator on an unmoving, static, bound field.

The same potential is the solution for the screening of an isolated electric charge inside a dielectric. In that case oppositely charged particles in the medium are shifted toward the isolated charge having the effect of smearing the charge out into a Yukawa-field charge density profile. This is because the opposite charges attract. What is shown in Fig. 3 is gravitational screening due to opposite gravitational fields that repel. Apparently, as the space-time fabric is concentrated toward the origin it does so at the expense of the surrounding space which is stretched. Both regions are stabilized by their self gravitation, but repelled from each other. The net effect is a region of positve and negative energy with zero total gravitational charge, but inertial mass, k.

7. *The Complete Relativistic Solution*

There are 3 ways to get this solution.

1. Add the interior solution to the exterior one.

   Gauge fields can simply be added. Adding the free relativistic (interior) solution and the far field (exterior) solution together, the necessary potential is obtained:

$$\Phi(r) = -m\frac{\left(1-e^{-kr}\right)}{r}. \tag{56}$$

2. Integrate the scalar field.

   For another interpretation of Eq. (56) consider the normalized scalar field ψ,

$$\psi(r) = \sqrt{\frac{k}{4\pi}}\frac{e^{-\frac{k}{2}r}}{r} \tag{57}$$

   Then the expected value of m within a sphere of radius, r, in such a state is

$$\begin{aligned} <m> &= \int_0^r \left(\psi(r)^\dagger m \psi(r)\right) 4\pi r^2 dr \\ &= m\left(1-e^{-kr}\right), \end{aligned} \tag{58}$$

   so that the Newtonian potential Φ as in Eq. (39) becomes

$$\Phi(r) = -\frac{<m>}{r} = -\frac{m\left(1-e^{-kr}\right)}{r}. \tag{59}$$

3. Use a Lagrangian density with an interaction term describing a mass interacting with its own gravitational field.

   Eq. (56) is also obtained by solving the K-G equation in the classical gravitational potential of the mass by adding an interaction term to the Lagrangian density for the K-G equation [23]. This is a material particle (region of spacetime) interacting with its own gravitational field with coupling constant k.

   It is this interaction that couples the inertial mass to the gravitational mass of matter.

   This works in an analogous way to the way the Higgs field couples to the electroweak field.

$$\mathcal{L} = \frac{1}{2}\Phi'(r)^2 - \frac{1}{2}k^2\Phi(r)^2 + \mathcal{L}_{\text{int}} \tag{60}$$

   with

$$\mathcal{L}_{\text{int}} = V(r)\,\Phi(r), \quad V(r) = -k^2\frac{m}{r} \tag{61}$$

   yielding

$$\Box\Phi(r) + k^2\Phi(r) - V(r) = 0 \tag{62a}$$

$$\Rightarrow \Phi(r) = -m\frac{\left(1-e^{-kr}\right)}{r}. \tag{62b}$$

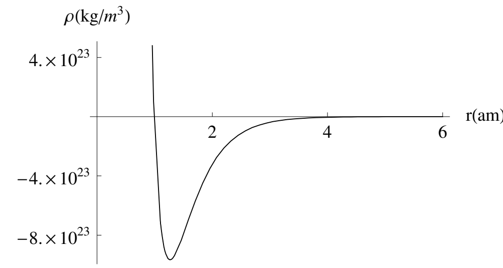

FIG. 5. Density as a function of radial distance for a neutral Higgs-like boson.

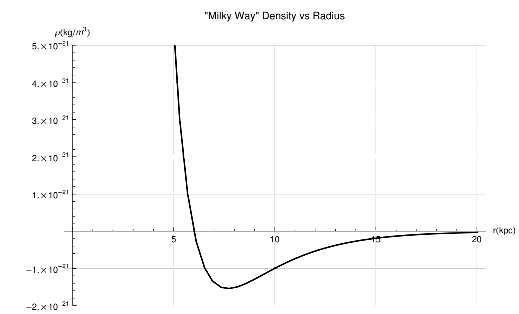

FIG. 6. Laue scalar for the complete gauge field for a Milky Way model ($m = 5.6\times10^{-5}\ kpc, k = 2.2/6)\ kpc^{-1}$).

Putting Eq. (56) into Eq. (43) gives the profile for the Laue scalar as shown in Fig. 5 or Fig. 6 for a Higgs-sized particle or a galaxy, respectively.

Putting the same potential into Eq. (45) gives,

$$\begin{aligned} M(r) = \frac{rc^2}{\kappa}\Bigg(mr\left(\frac{ke^{-kr}}{r} - \frac{1-e^{-kr}}{r^2}\right) \\ \times\, e^{-\frac{2m\left(1-e^{-kr}\right)}{r}} - e^{-\frac{2m\left(1-e^{-kr}\right)}{r}} + 1\Bigg), \end{aligned} \tag{63}$$

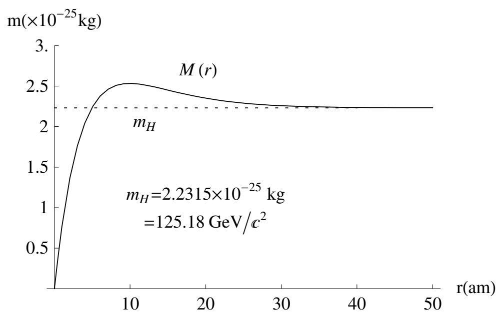

FIG. 7. Enclosed mass as a function of radial distance for a neutral Higgs-like boson.

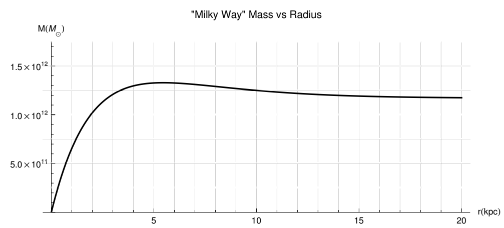

FIG. 8. Mass profile for the complete gauge field for a Milky Way model ($m = 5.6 \times 10^{-5}\ kpc, k = 2.2/6\ kpc^{-1}$).

the mass contained within a radius, r for the same 2 objects as shown in Fig. 7 or Fig. 8.

The limit as r$\rightarrow \infty$ is

$$\lim_{r\to\infty}(M(r)) = \frac{mc^2}{\kappa} = M. \tag{64}$$

a. Mass Eigenvalues The mass is also a parameter of the gauge field. It therefore gives rise to a non-linear eigenvalue equation. It is remarkable that those 2 masses are equal for any value of the mass or any value of the parameter k, in the case of a real scalar gauge field. That is consistent with the fact that all real scalar fields must solve the Klein-Gordon equation. It is an hypothesis that all particle masses of the standard model can be generated this way. This is true of both gauge fields and spinor fields. That should produce a spectrum of mass values for each algebraically distinct type of gauge field. The $SL(4,\mathbb{R})$ algebra, along with its inner and outer automorphisms, covers the Standard Model [11]. Therefore this is a methodology for determining the masses of the elementary particles from first principle.

Again from a quantum perspective, Eq. (56) gives

$$\Box\Phi + k^2\Phi - V = -\nabla^2\Phi + k^2\Phi - V = 0, \tag{65a}$$

$$\Rightarrow \hat{p}^2\Phi = V - k^2\Phi, \tag{65b}$$

so in the local frame the field momentum is still imaginary since k is real and V is negative.

Putting the potential, Eq. (56), into Eq. (40) and taking the limit shows that the metric tensor is finite at the origin as it should be.

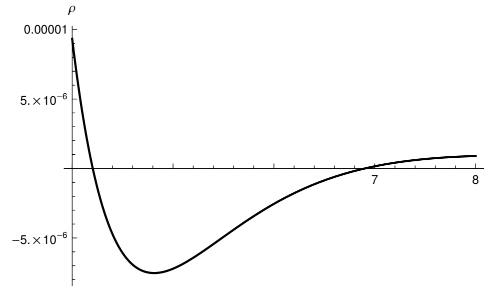

FIG. 9. Negative Laue scalar and crossing points for the complete potential in Eq. (56) ($\kappa = c = m = k = 1$).

$$\operatorname*{Limit}_{r\to 0}(g) = \begin{pmatrix} e^{-2mk} & 0 & 0 & 0 \\ 0 & -e^{2mk} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{66}$$

It was stated above that $\boldsymbol{A}$.1 and $\boldsymbol{A}$.2 were not entirely independent. Even if the goal here were not to eliminate singularities, Eq. (56) is the inevitable result of treating the classical potential as a GR gauge field. The Laue scalar and mass profiles are similar to those shown in Figs. 6 and 8; the density is still screened and goes through 0 and becomes negative - in this case decaying indefinitely.

Each mass is surrounded by a shell of negative energy in the case of a typical galaxy, giving rise to a bump in the mass curve, linked to the bump in the rotation curve through Renzo's Rule [24]. However, based upon the values of k and m, the density may not become negative, or cross the axis a second time in a positive-negative-positive pattern as shown in Fig. 9. The total mass remains the same while the singularity is eliminated. This is because the Yukawa term adds equal amounts of positive and negative energy.

This is scalar gravity returning [25], but now in tensor form that agrees with experiment, as shown below, wrapped in GR.

The Yukawa potential part of Eq. (56) represents a "nuclear" force while the other term, the classical gravitational potential per unit mass, -m/r, is just the residual. Note the nuclear part is repulsive; it prevents collapse. However, the covariant expression of this potential through the Einstein equation provides a more complicated picture.

That energy appears in covariant form as the Laue scalar, $T^{\alpha}_{\ \alpha}$.

So the mass is therefore composed completely of matter, or gravitational field, or spacetime(displacements); they are all equivalent satisfying $\boldsymbol{A}$.1.

This demonstrates the consequence of the theory. Previously the rule was that the right side of the EFE contains all non-gravitational sources of energy, hypothesized in covariant form. Now it contains *only* gravitational sources of energy, directly proportional to the covariant Einstein tensor.

This is in contrast to $\Phi(r) = \frac{1}{2}Log(1 - \frac{2m}{r})$ which gives the Schwarzschild solution. In that case the integrated mass is M, but it does not depend on r by Eq. (45). The Laue scalar is 0 and the solution is only valid outside the mass, which must show up as a discontinuity in the Laue scalar gradient at some point. That solution also allows singular black holes, where the entire mass is in gravitational field energy, but the field energy has no generally covariant form. Traditionally, non-covariant expressions have been derived for the gravitational field energy. In [26], for example, when such an expression is integrated over space (in isotropic coordinates), the total rest energy, Mc^2, is reached at $m/2$ instead of at 0. Such a result is bizarre, albeit non-covariant.

It has been argued that such a solution state exists far in the future for a collapsing body and is never actually attained in a finite time, as referenced from the outside. However it is still a solution, therefore allowed, and it is still singular.

8. *Applicability of the Solution*

The gauge potential in Eq. (56) is only applicable to regions with no strong or electroweak forces: black holes, galaxies in aggregate, spin 0 elementary particles and the exteriors of bulk matter. Although the weakest of forces, gravity works down to arbitrarily small scales; if enough energy is present in a small enough volume, it will collapse into a particle without singularity.

This solution is valid for masses of all sizes, provided the symmetry requirements of the metric tensor are satisfied. In particular this means that for normal aggregate matter like lead balls, stars and galaxies, the proper summations (integrals) are required as shown below.

9. *Aggregated Matter*

Assume for black holes

$$k = \frac{c^2}{\kappa\ m}, \tag{67}$$

the inverse geometric mass. Then the parameter, k, can be put in a form that covers all mass scales,

$$\frac{1}{k} = \frac{\hbar}{mc},\ m < m_p \tag{68a}$$

$$\frac{1}{k} = \frac{\kappa m}{c^2},\ m > m_p. \tag{68b}$$

$1/k$ is a minimum at the Planck mass. Below that the behavior is quantum and the wavelength increases with decreasing mass. Above that the characteristic length also increases but the wave nature is now hidden within the geometric mass and so the behavior is classical.

For the Equivalence Principle (EP) discussion below, the distinction will be made between the gravitational mass, m_g, and the inertial mass, m_i. In Eq. (50), m is clearly the gravitational mass. For quantum-domain fields

$$k = \frac{m_i c}{\hbar}, \tag{69}$$

the inverse Compton length. It derives from the free K-G equation so it is clearly (proportional to) the inertial mass. For elementary particle-sized masses this can be put into a more scalable form — that explicitly shows the coupling between gravitational and inertial masses:

$$\begin{gathered}\Phi(r) = -m\frac{\left(1 - e^{-kr}\right)}{r} = -\frac{m_g m_i}{{m_p}^2}\frac{\left(1 - e^{-kr}\right)}{kr}, \\ m_g = \frac{m\,c^2}{\kappa}, \quad m_i = \frac{\hbar\,k}{c}, \quad m_p = \sqrt{\frac{\hbar\,c}{\kappa}},\end{gathered} \tag{70}$$

m_p being the Planck mass.

The r-equation of geodesic motion for an initially stationary particle gives

$$\begin{gathered}\frac{d^2r}{dt^2} = -c^2 e^{-\mu\frac{4\left(1-e^{-kr}\right)}{kr}}\mu\left(\frac{1 - e^{-kr}}{kr^2} - \frac{e^{-kr}}{r}\right), \\ \mu = \frac{m_g m_i}{{m_p}^2}.\end{gathered} \tag{71}$$

If m_g is allowed to be negative, $m_g < 0$, then the acceleration is in the positive r-direction, that is, repulsive. This is assuming that a positive test mass travels in a positive timelike direction on the geodesics near m. Also of note is the asymmetry in the exponential factor for larger masses which falls off rapidly. This is shown in Fig. 10

For elementary particle-sized masses, $|m| << 10^{-8}kg$, this happens at many orders of magnitude below the particle wavelength due to the geometric mass in the exponent. The acceleration is symmetric with a limited cutoff.

For large masses, $|m| >> 10^{-8}kg$ with $m < 0$, the acceleration is unlimited for small r. Note that this refers to a large single mass, if such exists, not normal bulk matter made up of an aggregate of atoms. Aggregate matter does not present this behavior as shown below.

III. EXAMPLES

A. Gravitational Radiation

Take the tensor $\chi_{\mu\nu}$ in Eq. (32) to be symmetric, appropriate for a gravitational field, and solve Eq. (31) with base metric and coordinates as in Eq. (11). With

$$\chi_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \psi_1(t,x) & \eta_1(t,x) \\ 0 & 0 & \eta_2(t,x) & \psi_2(t,x) \end{pmatrix}, \tag{72}$$

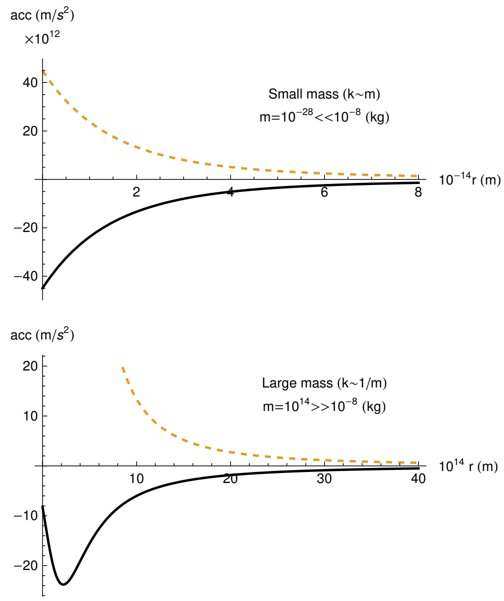

FIG. 10. Acceleration due to + and -(dashed) masses and asymmetry for larger masses.

Eq. (31) is solved since the y- and z-derivatives are applied to functions of t and x only. These functions need to solve the K-G equation,

$$\Box\chi_{\mu\nu} + k^2\chi_{\mu\nu} = 0, \tag{73}$$

with k=0 (m_i=0) for a wave solution (below k is wave number),

$$\begin{aligned} \Box\chi_{\mu\nu} = 0 \Rightarrow \chi_{\mu\nu} = \gamma\text{Sin}\left(k_\mu x^\mu + \alpha\right), \\ k^\mu = (\omega, k, 0, 0), k_\mu k^\mu = 0 \Rightarrow \omega = ck \end{aligned} \tag{74}$$

Also from Eq. (31) χ^μ_ν is traceless, and choosing ψ and η to have the same phase,

$$\begin{aligned} \chi^\mu_\mu = 0, \alpha = 0 \Rightarrow \psi_2 = -\psi_1 \\ , \eta_1 = \eta_2 = \gamma\text{Sin}\left(k_\mu x^\mu\right) \end{aligned} \tag{75}$$

giving

$$\begin{aligned} \chi^\mu_\nu = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \psi(t,x) & \eta(t,x) \\ 0 & 0 & \eta(t,x) & -\psi(t,x) \end{pmatrix}, \\ \psi(t,x) = \eta(t,x) = \gamma\text{Sin}\left(k_\mu x^\mu\right). \end{aligned} \tag{76}$$

Using the polarization matrices

$$P_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \tag{77}$$

these two solutions can be written as

$$\Phi_1 = \gamma P_1 \text{Sin}(\omega t - kx), \Phi_2 = \gamma P_3 \text{Sin}(\omega t - kx). \tag{78}$$

Since

$$P_1 \cdot \boldsymbol{k} = P_2 \cdot \boldsymbol{k} = 0, \tag{79}$$

these tensor waves are transverse as well as traceless (TT-gauge). Putting these TT-gauge fields into Eq. (16) yields

$$\begin{aligned} g_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -e^{2\eta} & 0 \\ 0 & 0 & 0 & -e^{-2\eta} \end{pmatrix}, \\ g_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -\text{Cosh}(2\eta) & -\text{Sinh}(2\eta) \\ 0 & 0 & -\text{Sinh}(2\eta) & -\text{Cosh}(2\eta) \end{pmatrix}. \end{aligned} \tag{80}$$

These metrics are very different from the ones obtained from the linearized theory, but agree to first order in γ. Putting either of these two metrics into the EFE gives the matter tensors

$$\begin{aligned} T_{\mu\nu} = \frac{\gamma^2\omega^2\text{Cos}^2(\omega t - kx)}{4\pi\kappa} \begin{pmatrix} -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \\ T^\mu_\nu = \frac{\gamma^2\omega^2\text{Cos}^2(\omega t - kx)}{4\pi\kappa} \begin{pmatrix} -1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \end{aligned} \tag{81}$$

Interestingly, $T_{\mu\nu}$ is the exact same result as the TT-gauge Isaacson pseudotensor [27] obtained from the linearized theory. As an aside, some alternate theories of gravity also give that Isaacson pseudotensor [28]. Now however it is a true covariant tensor and not limited to high frequencies for consistent interpretation. This is gravitational energy expressed in covariant form; no need for pseudotensors. This is only possible because now gravity is treated on the same footing as the other forces, as a gauge field. Matter/energy is created in a locale as the wave moves through compression/shear then destroyed. T^μ_ν is also traceless so that the negative energy density created is compensated for by an equal amount of negative pressure giving zero rest mass. The distortion appears to move but does not - only the wave, so it can transport the energy at the speed of light. This exemplifies that fact that gravitational energy has been moved to the right side of the equation (identity actually). This is achieved, in like manner with the other forces, by putting their gauge fields on the left side. This is an answer to whether gravitational radiation can carry or transfer energy. It does. Also it is a source of gravity as expected (G≠0). However EM radiation is not, as shown below.

B. Electromagnetic Radiation

Proceeding as above, take the antisymmetric tensor $f_{\mu\nu}$ in Eq. (32) only this time solve it for the vector equations Eq. (29) in charge- and current-free space where

$$f^{\mu\nu}_{;\nu} = j^{\mu} = 0. \tag{82}$$

This gives

$$\Box\phi^{\mu} = 0, \tag{83}$$

once again the K-G gauge equation for a massless field. Instead of a polarization tensor as above, there exists a polarization vector, **e**, with 2 possible orientations for transverse plane wave solutions.

$$\phi^1 = \boldsymbol{e}A\mathrm{Sin}\left(k_\mu x^\mu\right), k^\mu = \left(\frac{\omega}{c}, k_x, k_y, k_z\right),$$
$$k_\mu k^\mu = 0 \Rightarrow \omega = c|k| \tag{84}$$

$$e_y = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, e_z = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{85}$$

Taking the z-polarization for example,

$$\phi^\mu = (0, 0, 0, a\mathrm{Sin}(\omega t - kx)), \tag{86}$$

the EM field is, with $\chi(t,x) = ak\mathrm{Cos}(\omega t - kx))$,

$$F_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & \chi(t,x) \\ 0 & 0 & 0 & -\chi(t,x) \\ 0 & 0 & 0 & 0 \\ -\chi(t,x) & \chi(t,x) & 0 & 0 \end{pmatrix}, \tag{87}$$

and the gauge field is

$$\phi^{\mu}_{;\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \chi(t,x) & -\chi(t,x) & 0 & 0 \end{pmatrix}. \tag{88}$$

Putting this gauge field into Eq. (16) yields the metric,

$$g = \begin{pmatrix} 1 - \chi(t,x)^2 & \chi(t,x)^2 & 0 & \chi(t,x) \\ \chi(t,x)^2 & -1 - \chi(t,x)^2 & 0 & \chi(t,x) \\ 0 & 0 & -1 & 0 \\ -\chi(t,x) & \chi(t,x) & 0 & -1 \end{pmatrix}. \tag{89}$$

Putting this metric into the EFE gives the matter tensor,

$$G_{\mu\nu} = T_{\mu\nu} = 0. \tag{90}$$

The metric tensor shows changes in spacetime intervals as the wave passes, but the wave is not a source of gravity. Fig. (11) shows the spatio-temporal distortions along the plane-wave front for the eigenvalues of this metric. The shape of these curves indicate of a region of space rotating clockwise, then counterclockwise in the x-z plane as the wave propagates along the x-direction. The energy changes hand between the electric field(rotating) and the magnetic field(rotated). There is no curvature and no gravity because this is a traveling rotation - unlike the case for gravity waves where there is a traveling compression/shear.

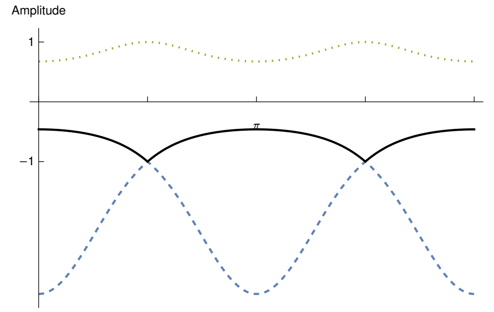

FIG. 11. Metric distortions of time (dotted) and space in the propagation direction (solid) and z (dashed).

This is a different gauge than in Eq. (24) where the metric was unchanged, but the result is the same - no curvature. This had to be true, since it was determined above that anti-matter generates anti-gravity and repels matter. If a system composed of an equal and symmetric amount of both, like positronium, created no gravity, then it would also be true for the photons that resulted from its annihilation. Both systems however still follow spacetime geodesics, so this result does not exempt EM radiation from the well documented redshift in a gravitational field of another mass or deflection by a massive object. What is interesting is that this result - that had to be true for logical consistency - arose automatically from the methodology. Although it is true for radiation fields of the gauges considered above, it is not true in general. Electromagnetic fields can be a source of gravity as shown below in the section on the the Aharonov-Bohm effect. The fact that the energy present does not cause curvature is explained by the fact that the electric field component contains positive energy and the magnetic field negative energy, also shown in the next section.

Historically, the EM stress-energy tensor is taken to be the matter tensor, T, asserting that the mass equivalent of the field energy is a gravitational source and therefore belongs on the right hand side of the EFE. The reason for this is the ansatz that T should contain all sources of energy, in an assumed covariant form, excepting of course gravitational energy. This is an unconfirmed assumption. All tests of GR involve gravitational fields created by masses. One implication of this is that the radiation component of cosmic expansion in ΛCDM models should set $\Omega\gamma = 0$, affecting cosmological time scales. Also, in the regime of comparable energy densities the evolution of the temperature will no longer be dictated by $\Omega\gamma$.

Equivalence Principle

This theory does not *require* $m < 0$ nor the gauge chosen above for the EM field. But if so, the equivalence principle would obviously need modification. If photons have no gravitational charge or if matter and antimatter repel each other, the ratio between gravitational mass and inertial mass would then take one of three values,

$$\frac{m_g}{m_i} = (-1, 0, 1) \tag{91}$$

for antimatter, EM radiation and matter, respectively. The sign changes would go with the gravitational mass; the inertial mass of the antiproton has already been measured to high accuracy and is in agreement with that of the proton [29]. Measurements of antihydrogen in Earth's gravitational field are ongoing at the LEAR project and have shown that antihydrogen falls down, toward Earth [30]. This requires explanation:

1. Strictly speaking, neither inertial nor gravitational masses are defined in GR [31]. Both arise from different applications of the Correspondence Principle: a simple Newtonian analysis does not suffice.
2. Consider a "dust" model, appropriate to non-interacting clouds of atoms. The divergence-less property of the Einstein tensor guarantees the density of such clouds drops out of the equations of motion. The motions are determined solely by the connections [32], which are dominated by Earth's gravity in this case. An anti-Earth would repel an Earth, or an anti-H would repel an H, for example. However, the gravitational potential gradient at the "surface" of a proton due to the Earth is roughly 10^8 times that of the proton itself. A small test mass follows geodesics regardless of its mass.

It is therefore predicted that an asymmetry would be measured in the free fall of H and anti-H at roughly 2 parts in 10^8.

C. Massive Charged Particle

Combining a static antisymmetric E-M field with a symmetric gravity field gives according to Eq. (32)

$$\boldsymbol{\zeta}_{\mu\nu} = \boldsymbol{\chi}_{\mu\nu} + \boldsymbol{f}_{\mu\nu}. \tag{92}$$

In spherical coordinates

$$\boldsymbol{\zeta}_{\mu\nu} = \begin{pmatrix} \Phi & -E & 0 & 0 \\ E & \Phi & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \boldsymbol{\zeta}^{\mu}_{\nu} = \begin{pmatrix} \Phi & -E & 0 & 0 \\ -E & -\Phi & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{93}$$

is the appropriate gauge for a spherically symmetric charged particle. It also corresponds to a complex scalar field as in Eq. (141) below. Putting this into Eq. (16) yields the metric tensor, g, which in turn gives the Einstein tensor, G. For $E(r) = 0$ the density and mass are the same as in Figs. 3 and 4. For $\Phi(r) = 0$ however, $G = 0$. This explains the observational fact that charged particles without gravitational mass do not exist.

D. The Aharonov-Bohm Effect

The above methodology can now be used to calculate the gravitational effects of electromagnetic fields and show their relationship to quantum theory. The Aharonov-Bohm Effect is a good example. It has been stated that this effect causes the vacuum to have structure [33] in that region which is free of magnetic field, but has a non-zero vector potential, $\mathbf{A}$. It will be shown that this "structure" is a displacement of events given by $\mathbf{A}$, whose unit is the meter as in Table 1. This displacement results in a spacetime shear and an associated gravitational field. The idealized experimental setup results in a clean separation between regions of space with EM fields and regions with a vector potential but no EM field.

Consider a long cylinder of radius, ρ, uniformly magnetized with magnetic field, $\mathbf{B}$, in the z-direction. The cylinder's mass, and gravitational field due to its mass, are ignored. The cylindrical coordinates, $\bar{x}^\mu$, and metric, $\bar{g}$, in the laboratory frame are

$$\bar{x}^\mu = (t, r, \theta, z), \bar{g} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{94}$$

The EM field and vector potential, Φ, are as follows. [33]

Inside the cylinder,

$$F_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & Br & 0 \\ 0 & -Br & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{95}$$
$$\Phi^\mu = \left(0, 0, \frac{B}{2}, 0\right),$$
$$\Phi_\mu = \left(0, 0, -\frac{Br^2}{2}, 0\right).$$

Outside the cylinder,

$$F_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \Phi^\mu = \left(0, 0, \frac{B\rho^2}{2r^2}, 0\right),$$
$$\Phi_\mu = \left(0, 0, -\frac{B\rho^2}{2}, 0\right). \tag{96}$$

The physical components of the vector potential outside are

$$\boldsymbol{A} = \left(0, 0, \frac{B\rho^2}{2r}, 0\right). \tag{97}$$

It has dimensions of length, and since its curl is zero outside it can be written as the gradient of some scalar function, χ:

$$\boldsymbol{\nabla} \times \boldsymbol{A} = \boldsymbol{\nabla} \times \boldsymbol{\nabla}\chi = 0 \Rightarrow \chi = \frac{B\,\rho^2}{2}\theta. \tag{98}$$

Using Eq. (17) to change to the dimensionless field, b, outside the gauge field is given by,

$$\phi^{\mu}_{\ ;\nu} = \zeta^{\mu}_{\ \nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{b\rho^2}{2r} & 0 \\ 0 & -\frac{b\rho^2}{2r^3} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{99}$$

and the metric tensor is given by Eq. (16),

$$\begin{aligned} g_{\mu\nu} &= \left(e^{\boldsymbol{\zeta}}\right)^{\mu}_{\ \lambda} \bar{g}_{\mu\nu} \left(e^{\boldsymbol{\zeta}}\right)^{\nu}_{\ \tau} \\ &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\text{Cosh}\left(\frac{b\rho^2}{r^2}\right) & r\text{Sinh}\left(\frac{b\rho^2}{r^2}\right) & 0 \\ 0 & r\text{Sinh}\left(\frac{b\rho^2}{r^2}\right) & -r^2\text{Cosh}\left(\frac{b\rho^2}{r^2}\right) & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \end{aligned} \tag{100}$$

This is the metric for a shear in the r-θ plane with a maximum value at the cylinder boundary, r=ρ.

From the EFE, Eq. (33),

$$T^{\mu}_{\ \nu} = \begin{pmatrix} -\frac{b^2\rho^4c^2\text{Cosh}\left(\frac{b\rho^2}{r^2}\right)}{4\pi r^6\kappa} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{b^2\rho^4c^2\text{Cosh}\left(\frac{b\rho^2}{r^2}\right)}{4\pi r^6\kappa} \end{pmatrix}. \tag{101}$$

There is therefore gravitational energy outside the magnet due to the magnetic field inside the magnet. There is a negative energy density, and positive stress in the z-direction equal in magnitude to the density. T is not traceless, and the trace is negative. This means there would be a repulsive gravitational force in the r-direction in the region that is outside the magnet and outside most of the gravitational field. Integrating the Laue scalar over the exterior region, and multiplying by c^2 gives the total energy:

$$\begin{aligned} &\int_{\rho}^{\infty} T^{\mu}_{\ \mu} c^2 \sqrt{-\|g\|} 2\pi z dr \\ &= \frac{z\,c^4(-1+\text{Cosh}(b) - b\,\text{Sinh}(b))}{2\,\kappa}. \end{aligned} \tag{102}$$

$$(1 - \text{Cosh}(b) + b\,\text{Sinh}(b)) = \frac{b^2}{2} + O(b)^4 \tag{103}$$

Historically it was the fact that the EM field could store energy that argued for its reality. Here it is stored in the gravitational field as spacetime displacements; another reason no additional fields are necessary.

The gravitational energy stored does not depend at all on ρ, the radius of the magnet. This is because the space of a cross-sectional disk of the magnet is rotated as a unit with no shears within. The space outside has a higher energy density Eq. (101) with increasing ρ, but that is exactly offset by the fact that there is less of it. That is, the bottom limit of the integral increases. All of the energy of the magnetic field does not serve as a source of gravity; T is 0 inside. The outside energy must arise as some of the work needed to establish the field.

Picture for Quantum Theory

This gives quantum fields a "picture". **A** is a physical displacement of points in space. In the covariant derivative

$$\hat{p} \to (\hat{p} - e\,\boldsymbol{A}), \tag{104}$$

the momentum operator, $\hat{p}$, is the generator of translations. The covariant derivative indicates that the translation has to be shifted to compensate for the physical displacement of spacetime by the vector potential, **A**, to obtain the net translation [34]. Incidentally, the zeroth component of the covariant form of the covariant derivative,

$$\hat{p}^0 \to \left(\hat{p}^0 - e\,\boldsymbol{A}^0\right), \tag{105}$$

indicates the energy operator as the generator of temporal translations with the shift due to the scalar potential. This is consistent with the above description, Eq. (21), of the electric field as the gradient of a time translation. So it is clear than uncertainties in $\boldsymbol{A}$ inherit directly from uncertainties in the coordinates so their commutation relations inherit as well.
Concomitantly the quantum field undergoes a local gauge transformation,

$$\Psi \to e^{i\,\Lambda}\,\Psi = e^{\frac{i\,e\,\chi}{\hbar}}\,\Psi, \tag{106}$$

so that the phase angle, Λ, is

$$\Lambda = \frac{e\,B\,\rho^2}{2\,\hbar}\theta = \frac{B\,\pi\,\rho^2}{h/e}\theta = \frac{n}{2}\,\theta, \tag{107}$$

where n is the number of quanta of magnetic flux through the cylinder.

It is usually stated that Λ is a rotation in the "internal space" of the field. Here it is clear that the displacements are spatial and these are real, physical rotations in the r-θ plane. Until now, solutions to the K-G equation were used above for event displacements regardless of whether or not they were macroscopic fields or quantum fields. That is because it does not matter. This point of view is a consequence of $\boldsymbol{A}$.1. This example shows that quantum fields are therefore made of the same "stuff" as spacetime, or gravitational fields, etc. The only difference is that for small scales the fields need to be treated as operators and measurement theory comes into play. Besides, since there is no prevailing "picture" of quantum fields, viewing them as spacetime amplitudes, densities, displacements, etc. cannot matter as long as they obey the same equations. However, this can provide a great insight into their nature and connection to classical theory and unification. The key foundational point is that the equations are on gravitational gauge fields (in the sense of Eq. (32)); they are flat space equations. Their incorporation into GR is through Eq. (16). EM has served as a bridge between GR and QT.

Of course the gauge transformation, Eq. (106) is on a complex function, Ψ. There is nothing special about using the complex numbers in quantum theory. Eq. (106) is exactly equivalent to

$$\Psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix} \rightarrow \begin{pmatrix} \mathrm{Cos}(\Lambda) & -\mathrm{Sin}(\Lambda) \\ \mathrm{Sin}(\Lambda) & \mathrm{Cos}(\Lambda) \end{pmatrix} \begin{pmatrix} \phi \\ \chi \end{pmatrix} = e^{i\Lambda}\Psi \tag{108}$$

separating the complex equation into coupled real equations.

E. Galaxies

The most compelling evidence for this model of gravity comes from galaxies. This one simple model gives Dark Matter(DM), Dark Energy(DE), Baryonic Tully-Fisher Relation(BTFR), Renzo's Rule, (Modified Newtonian Gravity(MOND), HSB and LSB galaxies and the Universal Rotation Curve(URC).

1. *The Geodesics*

The following are geodesic equations of the metric in Eq. (40) for circular orbits where $v = \omega\, r,\ dr = 0$.

$$\frac{d^2t}{ds^2} = -2\frac{dt}{ds}\frac{dr}{ds}\Phi'(r) = 0,\ \frac{dt}{ds} = \gamma, \tag{109a}$$

$$\gamma = \frac{1}{\sqrt{e^{2\Phi(r)} - \frac{r^2\omega^2}{c^2}}} = \frac{1}{\sqrt{e^{2\Phi(r)} - \frac{v^2}{c^2}}}, \tag{109b}$$

$$\frac{d^2r}{ds^2} = \gamma^2\frac{d^2r}{dt^2} = \frac{\gamma^2 v^2 e^{2\Phi(r)}}{rc^2} - \gamma^2 e^{4\Phi(r)}\Phi'(r) = 0, \tag{109c}$$

$$v^2 = r\, c^2 e^{2\Phi(r)}\Phi'(r) \tag{109d}$$

2. *Baryonic Tully-Fisher Relation*

Combining Eq. (45) and Eq. (109d) we see that

$$c^2\left(1 - e^{2\Phi(r)}\right) - \frac{\kappa M(r)}{r} = c^2 e^{2\Phi(r)} r\Phi'(r) = v^2. \tag{110}$$

Interpreting the velocity, v, as the speed of the circular orbits of the baryons, the terms on the left of Eq. (110) can be thought of the total matter minus the DM which is the total baryonic matter. There is no DM per se, only a covariant gravitational field within which the baryonic matter is embedded. In approximation for small $\Phi(r)$ it is

$$c^2\left(1 - e^{2\Phi(r)}\right) - \frac{\kappa M(r)}{r} \approx \frac{2\kappa M(r)}{r} - \frac{\kappa M(r)}{r} = \frac{\kappa M_b(r)}{r} = v^2 \tag{111}$$

Squaring both sides of Eq. (111) gives

$$\kappa M_b(r)\left(\frac{\kappa M_b(r)}{r^2}\right) = v^4 \tag{112}$$

This starts to happen when $r \approx \mu/k$ and the rotation curve starts to flatten out. At this point the Newtonian gravitational acceleration is at the MOND value, $a = 1.24 \pm .14 \times 10^{-10} ms^{-2}$ [35],

$$\kappa M_b\left(\frac{k^2\kappa M_b}{\mu^2}\right) = v_f^4, \tag{113}$$

and

$$\kappa M_b = A v_f^4,\ A = .8/a, \tag{114}$$

the BTFR.

3. *Universal Rotation Curve*

The Universal Rotation Curve (URC) of spiral galaxies is a method of normalizing the shape of the curves by a scale length [36]. In this current model of rotation curves it is simple. The potential

$$\frac{me^{-kr}}{r} = \frac{kme^{-kr}}{kr} = \frac{\mu e^{-u}}{u},\ u = kr,\ \mu = km \tag{115}$$

4. *Dark Matter*

The K-G equation is linear in the gauge fields so they can be added for aggregates of baryonic matter like a galaxy. The mass profile in Fig. 8 is similar to a typical galaxy rotation curve. This explains the puzzle of the correlation between total baryonic matter/luminosity and DM. DM is just the normal gravitational field of the galaxy as a whole plus the aggregate baryonic matter when the gauge potential is given by Eq. (56). The flattening of the rotation curve is due not to extra DM, but to the negative energy shell of the gravitational field of the normal baryonic matter. The rotation curve of the Milky Way as shown in Fig. 12 is patterned after that given in [37]. As has been shown, it is not necessary to

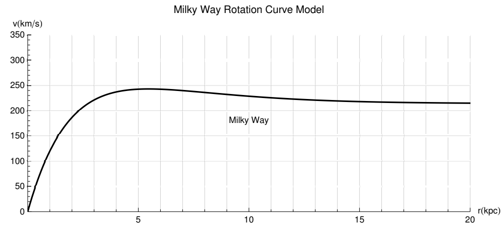

FIG. 12. Rotation curve model of the Milky Way.

"engineer" rotation curves as is normally done. Nevertheless, it can be done that way using the potential as given by Eq. (56) rather than the popular NFW density profile.

The potential in Eq. (56) is matter whether baryonic or not. The K-G equation is linear in the gauge fields so they can be added for aggregates of baryonic matter. For non-baryonic matter, it is just pure uncondensed matter, massive covariant gravitational field, or Dark Matter. Consider a solution corresponding to the baryonic matter in a typical galaxy, embedded in a larger mass of non-baryonic matter (Dark Matter):

$$\Phi(r) = -m^{+}\frac{\left(1-e^{-k^{+}r}\right)}{r} - m^{-}\frac{\left(1-e^{-k^{-}r}\right)}{r}. \quad (116)$$

Assuming circular orbits [38], the r equation of motion for the metric in Eq. (40) is

$$\frac{d^2r}{ds^2} = \frac{e^{2\Phi(r)}r\omega^2}{c^2} - e^{4\Phi(r)}\ \Phi'(r) = 0. \quad (117)$$

giving

$$v^2 = r^2\omega^2 = c^2re^{2\Phi(r)}\Phi'(r). \quad (118)$$

With the potential in Eq. (121), Fig. 13 shows generally qualitative fits for this gauge field to actual rotation curve data for the galaxies M33 and NGC 4157, assuming a baryonic mass content of $20 \times 10^9 M_{\odot}$and a diameter of 60,000 LightYears for M33. The NGC 4157 curve is fit.

The typical flat velocity profile is seen farther out on the spiral. It is common that galactic rotation curves are similar to the one for NGC 4157 in that there is a dip after the first peak before the slope increases again. It is due to the dark matter taking over from the more central baryonic matter. From simple Newtonian mechanics any matter distribution that increases linearly from the origin (inverse square density) gives circular orbits with constant speed. The mass distribution in Fig. 2, for example, is approximately linear over a wide range of radii. However, this is not the source of linearity here.

Most DM models use the TOV equation, but in the Newtonian limit, such as [39]. If the potential in Eq. (56) were just treated as Newtonian, none of the interesting features of the mass profiles would obtain: asymptotic flatness, humps and dips, mass discrepancies, non-Newtonian behavior and others. These are GR effects.

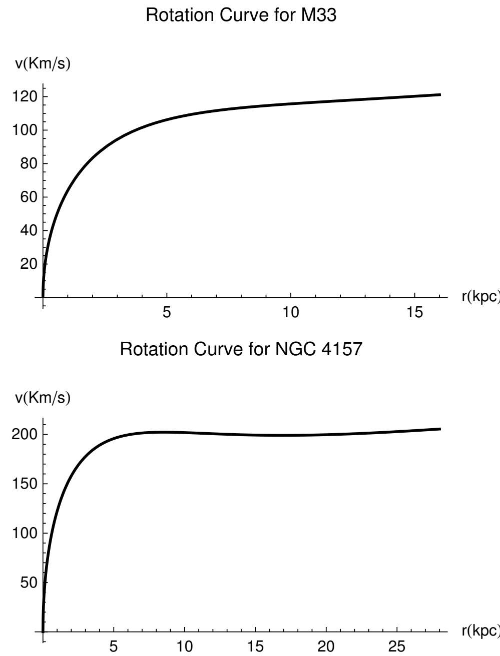

FIG. 13. The potential in Eq. (121) provides a qualitative fit for actual data from M33 and NGC 4157.

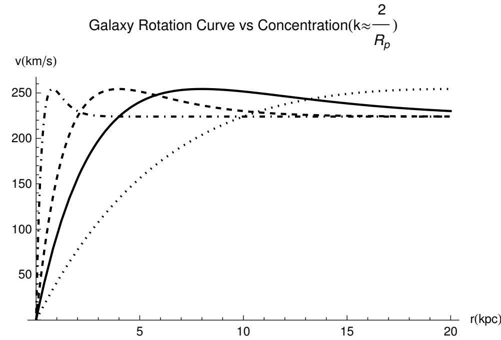

FIG. 14. Rotation vs Concentration.

Most importantly it solves the puzzle of the connection of the excess mass with light. The DM is the mass of the (covariant) gravitational field attached to the visible (baryonic) mass. This is currently the strongest "evidence" for this gauge field hypothesis.

It is clear that the stress-energy does not describe a Newtonian fluid: the radial pressure is different from the two orthogonal stresses. The TOV equation is not appropriate, but this is what is to be expected from a spherically symmetric, static DM halo [39].

Under Renzo's Rule [24], the mass profiles mimic the rotation curves. Therefore these mass profiles show that all the various galaxy morphologies can be described by just the one potential in Eq. (56). Its 2 parameters, m and k, map to v_f and the concentration respectively. Large ks correspond to HSB galaxies, small ones to LSB galaxies with all types in between. It can also map to the double-humped morphology which is indicative of a

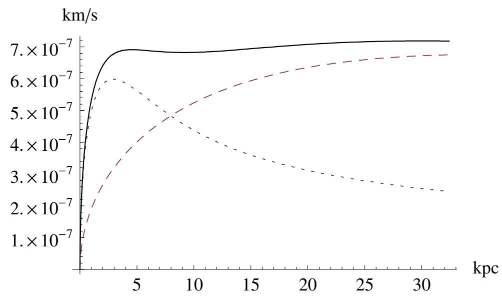

FIG. 15. Rotation curve showing flatness due to offsetting contributions from baryonic (dotted) and DM (dashed) components.

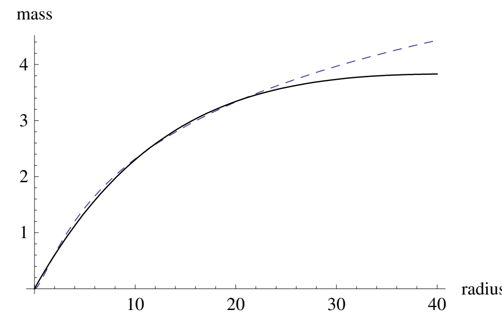

FIG. 16. Comparison of integrated density profiles for theory (solid) and NFW (dashed).

bulge [24]. In this case, 2 exponential terms with different ks are needed. That hints at bulges arising from mergers with the original halo intact.

The $\frac{(1-e^{-kr})}{r}$ feature of DM halo potentials provides a natural core. This avoids the cuspy halo problem that plagues traditional halo models for HSB galaxies. Applied to the LSB DDO 154, it predicts a flattening of the rotation curve above 10 kpc. measurements have shown a dip around 8 kpc leading to the conclusion that the edge had been reached [40]. This model indicates the dip is merely the pre-flat hump common to many galaxies.

Fig. 15 shows rotation curves for the baryonic matter alone, the DM alone and their combination. The flat velocity profile is seen to be due to an increasing contribution from DM which precisely offsets a decreasing profile from the baryonic matter. This fine-tuning can be obtained by adjusting the curvature parameter, k, as well as the dark matter ratio. Fine-tuning is also used in the parameters of the NFW density profiles usually used to obtain matches also [41]. There is a big difference here though. The NFW is a purely phenomenological profile used in simulations to achieve the necessary density profiles. Here, though, there is no choice of model; it is dictated by theory.

Fig. 16 shows a comparison of the integrated mass from this theory-based density profile to one based on NFW. Unlike the NFW model, no arbitrary cutoff at some high virial radius is needed because the density proceeds exponentially to zero on its own. In addition at low radii the increasing density, as shown above, can be integrated all the way to the origin so that no low cutoff is needed either. These facts make the theory-based density profiles superior to NFW. It should also be mentioned that since the NFW model is used extensively in DM gravitational simulations, the potentials of Eq. (56) or Eq. (121) is superior there as well. They are the actual theory-based gravitational potentials and they behave very well in simulations, having finite values at arbitrarily small separations, overlaps and superpositions.

5. *Dark Energy*

It is hardly a coincidence that DE came to prominence at an era when galaxy clustering was mature. As was shown, the concentration of DM, so to speak, within large gravitational structures stretches the exterior space into a negative energy state. This is a consequence of the tracelessness of $\mathfrak{sl}(4,\mathbb{R})$.

6. *Black Hole vs Red Hole*

Note that although spacetime is extremely distorted for $r \approx 2m$, there is no event horizon and no singular black hole. Such a black hole without singularity will be called a red hole for convenience. It appears that all the current observational evidence for black holes appears to be consistent with red holes as well, but they should be distinguishable for large enough fields. Any phenomenon occurring at r_b around a black hole occurs at r_r around a red hole, a smaller radius.

For example, Fig. 17 shows the differential redshift between a black hole with Schwarzschild radius, r_s, and a red hole of the same mass, $r_s/2$. 400 nm light emitted at the Innermost Stable Circular Orbit (ISCO), $r = 3r_s$, would be redshifted to about 490 nm for a black hole. For a red hole the same 490 nm red shift occurs at about $r = 2.45r_s$, a somewhat smaller radius. A model independent method of measuring the mass of the hole and the radius of the accretion disk at the ISCO should be able to distinguish the black hole model from the red hole model.

In terms of geodetic effects in high fields the two models have a relative difference of about 1% at $r \sim 7.5\, r_s$ and 10% at $r \sim 2.5\, r_s$ as shown in Fig. 18.

The concentration parameter, k, is set to $k = 1/m$ for black holes. Higher k values have a negligible effect of the model differences. However, lower concentrations for the same mass exhibit progressively larger differences. If such objects exist they could be excellent candidates for testing the model.

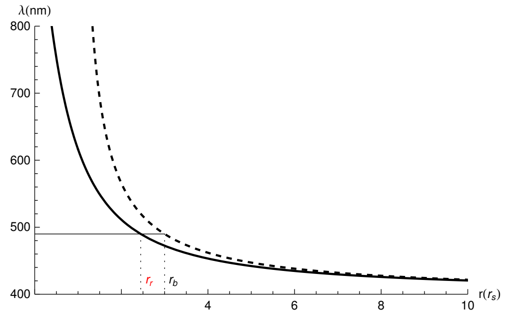

FIG. 17. ISCO red shift of 400 nm light, black hole (dotted) vs. red hole models.

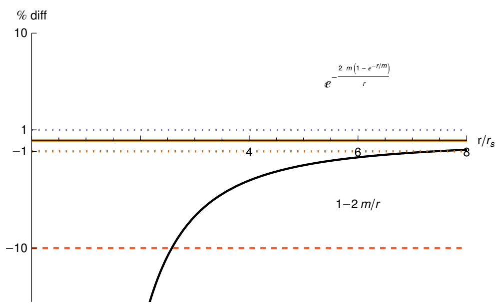

FIG. 18. % difference between standard black hole metric potential relative to red hole potential.

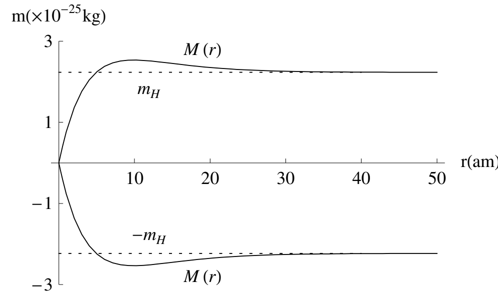

FIG. 19. Particle and anti-particle. A real particle would oscillate between these two configurations.

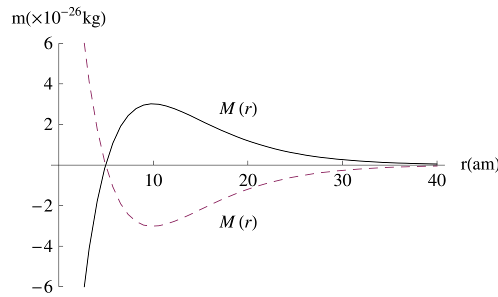

FIG. 20. Particle and anti-particle. A real particle would oscillate between these two configurations.

7. *Galactic Shields*

IV. CONSEQUENCES

A. The Nature of the Vacuum

"Empty" space is therefore composed of the tail ends of all the matter in the universe. Given the dynamic nature of the universe, spacetime events undergo constant displacement fluctuations; these are the vacuum fluctuations. Each fluctuation has equal positive and negative energy density regions of the Yukawa gauge potential under GR. By themselves they do not contribute to the a Cosmological Constant. This explains the ≈ 120 order-of-magnitude Cosmological Constant Problem.

That the matter of the universe is spread throughout spacetime is likely why the ΛCDM model works so well — in spite of the clumpiness of large scale structure.

B. Wave Nature of Matter

The above clearly shows that the wavefunction of a particle is *not* an epistemic phenomenon. A particle is a continuum, spread over a finite region of space. The parameter, k, is a measure of how spread out it is. This is in line with the ontic interpretation of the PBR theorem. This has been partially obscured by the use of i in the wavefunction. That shows the Schrödinger equation, for example, as a pair of coupled real equations; a physical displacement of spacetime events. This metric in Eq. (66) of course precludes the notion of a "point particle". At a small enough scale particles are just a continuum field.

1. *Elementary Particles*

Figures Figs. 5 and 7 show the density and mass profiles of a static, neutral spinless particle with the mass, m_H, of the Higgs boson. They depict a particle with a positive energy core having an energy greater than the particle's total mass-energy. This is enclosed by a negative energy shell compensating the excess energy of the core. The result is a total mass m_H. Of course a real Higgs particle will not be zero energy. As mentioned above the gauge field is oscillatory as in Eq. (52) giving the above profiles as standing waves, Fig. 19. It is its own antiparticle. A Lorentz transformation gives this as a traveling wave when the particle momentum is non-zero and the field momentum is real. Unseen in Fig. 20, both curves start at the origin at a scale of $\sim 10^{-54}\, m$.

2. *Locality*

For $r >> 2m$ the small residual mass density as seen from the laboratory frame is the mass due to the energy of what is usually referred to as the external gravitational field. However, it is the extension of the mass, through space, that interacts directly with another mass; one mass melds smoothly into another. As noted above, if they interact without fields, then they must not be at a distance. Their gauge fields superpose being linear K-G equation solutions, but the stress-energy tensors do not, being non-linear in the metric. This means that the "whole" is different than the sum of parts. Two separate masses are in a sense one thing. In other words they are entangled, or summed states.

3. Comparison With Experiment

The precision measurements of the geodetic effect by Gravity Probe B is in agreement with theory to better than 0.5% [42]. In particular the traditional expression for the geodetic orbital precession of the on-board gyros is given by [43]

$$\begin{aligned}\Delta\alpha &= -2\pi\left(-1+\sqrt{1-\frac{2m}{r}-\frac{m}{r}}\right)\text{Sin}[\theta]\\ &\approx \frac{3m\pi\text{Sin}[\theta]}{r}+\frac{9m^2\pi\text{Sin}[\theta]}{4r^2}\end{aligned} \tag{119}$$

while here we have ($k \approx \infty$ since this is an aggregate of atoms) for the 650 km high orbit),

$$\begin{aligned}\Delta\alpha &= -2\pi\left(-1+\sqrt{e^{-\frac{2m}{r}}-\frac{m}{r}}\right)\text{Sin}[\theta]\\ &\approx \frac{3m\pi\text{Sin}[\theta]}{r}+\frac{m^2\pi\text{Sin}[\theta]}{4r^2}\end{aligned} \tag{120}$$

These expressions agree to first order in m/r. They disagree to second order by a factor of 9, but the magnitude of that term is one part in 10^{-10} and 10^{-9}, respectively, of the first term. These differences are clearly beyond the capabilities of that experiment. So this theory is not inconsistent with the traditional formulation in neither the case of the black holes, nor the weak gravitational field of the Earth.

4. Dark Matter

Macroscopic fields may be realized in two ways. Macroscopic baryonic fields can arise from large masses like stars, gas and planets. In this case the geometric mass arises from the sum (appropriate integral) of all the constituent elementary particles. Their gauge fields can be added due to the linearity of the K-G equation. Also there is nothing to require that k comes from elementary particles at all. As long as it is a solution of the K-G equation it can represent nothing more than a displacement of events over some region specified by k. In this case $k \neq \frac{mc}{\hbar}$ and $k \neq \frac{c^2}{\kappa m}$. This fits the properties of Dark Matter exactly. No dark matter particles have yet been detected [44]. It is likely that at some critical density DM condenses into elementary particles. This may correspond to the well-documented acceleration scale found in the outer regions of spiral galaxies, the energy density being proportional to the square of the acceleration. Eq. (56) can also be considered as the limiting case of the superposition of 2 free K-G gauge solutions of this type - one of positive gravitational mass and one of negative gravitational mass:

$$\Phi(r) = -m\frac{\left(e^{-k^- r}-e^{-k^+ r}\right)}{r}. \tag{121}$$

As k^- becomes arbitrarily small, Eq. (121) approaches Eq. (56), and all of its features. It looks like a non-singular mass at the origin whose energy is entirely gravitational and covariant and is balanced at very large distances by an equal and opposite mass of negative gravitational energy. Such a configuration can be created from nothingness (not the vacuum) without energy in the "free lunch" scenario. This will be taken to its logical conclusion in the cosmology section below. This is a case of very long wavelengths.

C. Quantization of Charge

Since the EM field in (30) is a gauge field, it is a gravitational potential. In the traditional Reissner-Nordström metric [45] the contribution, Φ_g, from the electric field, E, is proportional to q^2 so that

$$g_{00} = 1-\frac{2m}{r}+\Phi_g = g_{00} = 1-\frac{2m}{r}+\frac{Cq^2}{c^2r^2}. \tag{122}$$

Here Φ_g can be viewed as the EM self-energy, and as a source for the gravitational field,

$$\frac{Cq^2}{c^2r^2} = \frac{Cq^2/\left(rc^2\right)}{r} = \frac{Cm_e}{r}, \tag{123}$$

with $m_e$the mass equivalent of the EM energy. However, if the gauge field is now proportional to the electric field,

$$\Phi_g \propto \frac{Cq}{r^2} = \frac{Cq/r}{r}, \tag{124}$$

then the constant, C, must contain a factor of q, even if only on dimensional grounds. Since C is constant and the energy is proportional to q^2, then there has to be a universal q, equal to e, the basic electronic charge, contained in C. In fact

$$C = \frac{\kappa e}{4\pi\epsilon_0 c^4} \tag{125}$$

in SI units. So quantization of charge is a consequence of identifying the EM field as a GR gauge field.

D. Higgs Field

The physical reason for the Yukawa potential Eq. (50) in Eq. (56) is to prevent the singularity. That is why it is attached to all massive particles. This is the classical analog of the Higgs boson. It is a solution to the scalar field equation and it provides all massive particles with inertial mass, k. It has equal and opposite positive and negative energy parts, like the particle-antiparticle pairs from Higgs decays [46]. Also it has zero energy, which is lower than the vacuum, like the Higgs. From a quantum perspective this is an imaginary-mass field like the Higgs (Eq. (55b)).

Spacetime contains matter and therefore energy; it corresponds to the Higgs field in quantum terminology. It provides a "picture" for it. If enough energy density is present at some event, it will collapse into a gravitationally bound structure. This structure then shields itself from becoming a singularity by pulling a "Higgs" from spacetime (the vacuum). In other words, it is more energetically favorable to form a gauge solution to the K-G equation than it is to form a singularity; the singularity does not solve the K-G equation locally, a necessity of relativity. This "Higgs" is in fact the source of the inertial mass of particles, coupled as it is to spacetime. The Higgs field that permeates all space is simply spacetime itself, which is not empty; it contains solutions to the K-G equation as gauge fields.

E. Cosmology

Boundary Condition

Until now it has been assumed that fields came from small displacements of spacetime from its value without the disturbance, that is, far from the disturbance. Nothing has been said about the boundary condition, assuming space was Lorentzian at large distances. As mentioned above $G_{\mu\nu} = 0$ is wrong, at least near the source. Although G=0 still means the space is empty, this condition is not to be found, except at the boundary between equal and symmetric distributions of matter and antimatter beyond the observational horizon of our universe. As a solution of gauge type Eq. (56) from a star decays, it blends into the dark matter of the galaxy as in Eq. (121), which blends into the dark matter of the local group, and so on. The metric decreases exponentially from the source never exactly reaching zero. This suggests that the boundary condition on the metric for a localized matter distribution approaches a small nonzero "vacuum" value corresponding to Eq. (121) for its encompassing distribution. Proof of this is that all aggregations of matter measured contain DM. There are some dark galaxies but recent re-calibrations show that at least one galaxy thought to be devoid of DM does indeed contain it. So the metric for any distribution of matter will tend at large distances to the gauge field solution, Eq. (56) for the larger distribution of matter in which it participates, that is, its dark matter.

ΛCDM models estimate about 16% of matter is baryonic, the rest being Dark Matter. It is an obvious speculation to consider that the baryonic matter condensed from the DM the same way a cloud forms from water vapor at a critical pressure/energy density. From Eq. (41)

$$T^0_{\ 0} = \rho = T^1_{\ 1} = -P \qquad (126)$$

so that the ratio of radial pressure to rest density is -1, giving these solutions the same property as a Cosmological Constant, Λ, except that it can vary in both space and time. These solutions however occur at any scale, so they can represent the DM as shown above for galaxies, galaxy clusters, superclusters, etc. They have negative pressure and have regions of negative energy density in their outer regions, either of which may appear as Dark Energy fueling the accelerated expansion of the cosmos. In addition, time dependent solutions like Eq. (52) may mimic Quintessence. So at once these "scalar fields" may provide the seeds for large structure formation while accounting for both DM and DE obviating the need for Λ, inflation or other heretofore unobserved phenomena. That is, these are just normal gravitational fields expressed as gauge fields.

Critical Energy Density (Mass Discrepancy-Acceleration Relation)

The pattern of Eq. (56) repeats itself at all scales. At some point it appears that spacetime collapses into a gravitationally bound structure. The classical energy density near the "edge", R for a mass M is

$$\mathcal{E} = \frac{1}{8\pi\kappa}\left(\frac{\kappa M}{R^2}\right)^2 \propto \left(\frac{M}{R^2}\right)^2 . \qquad (127)$$

Very roughly, M/R^2 in SI units for a neutral meson, a galaxy and the observable universe might be

$$\frac{10^{-28}}{(10^{-14})^2} \sim \frac{10^{42}}{(10^{21})^2} \sim \frac{10^{52}}{(10^{26})^2} \sim 1 \qquad (128)$$

Although this is a crude estimate, it is interesting that over such an enormous scale the ratio is about the same. That leads to the speculation that structure formed from the outside in. That gives an acceleration at the boundary of

$$a = \kappa\frac{M}{R^2} \sim .667 \times 10^{-10} m/s^2 \qquad (129)$$

which is less than a factor of 2 from the value $1.2 \times 10^{-10} m/s^2$ which is the small acceleration cutoff value [35] for the MOND model. The curvature parameter, k, in these potentials determines where the zero energy, zero scalar curvature radii are located. It is at these radii where the Mass Discrepancy problem begins; it is where the rotation curve profiles flatten out. k is proportional to the mass enclosed. It is possible that when these metrics for matter distributions are made dynamic they will provide a calculation for both critical energy density and the low acceleration threshold.

Lithium Problem

Baryon Asymmetry

As shown above, there is some asymmetry between matter and anitmatter, although their masses are the same. Consider the g_{00} component of the Schwarzschild metric (which is still valid for T=0) with M replaced by -M.

$$g_{00} = 1 - \frac{2\kappa M}{c^2 r} \to 1 + \frac{2\kappa M}{c^2 r} \tag{130}$$

This is obviously not symmetric - one expression can approach zero and the other cannot. However the replacement

$$1-\frac{2\kappa M}{c^2 r} \to -1+\frac{2\kappa M}{c^2 r} = -\left(1 - \frac{2\kappa M}{c^2 r}\right) = -g_{00}. \tag{131}$$

restores symmetry if g→-g. This amounts to switching from the Mostly Minus (MM) convention to the Mostly Plus (MP) convention for the metric. The prevalent way to do this is to concomitantly change the matter tensor T→-T in the EFE. In this already-unified field theory the EFE is postulated as an identity so the actual matter tensor changes sign, not the equation.

Since matter and antimatter are mutually repulsive it is obvious where the missing antimatter is. It separated from matter in the early epochs of the universe and is still out there beyond the horizon This suggests the interpretation that a change in metric signature changes from a region of space dominated by matter to one dominated by antimatter. That would also imply that g=0 at the boundary. The boundary then is the region where there is no matter and no spacetime. This restores the sound philosophical principle that space and time rely on matter for their existence. This was Einstein's belief as a consequence of Mach's principle [10].

For example, the static, spherically symmetric gauge field in Eq. (38) changes

$$\zeta_{\mu\nu} = \begin{pmatrix} \Phi & 0 & 0 & 0 \\ 0 & \Phi & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} -\Phi & 0 & 0 & 0 \\ 0 & -\Phi & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\zeta^{\mu}_{\nu} = \begin{pmatrix} \Phi & 0 & 0 & 0 \\ 0 & -\Phi & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} \Phi & 0 & 0 & 0 \\ 0 & -\Phi & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{132}$$

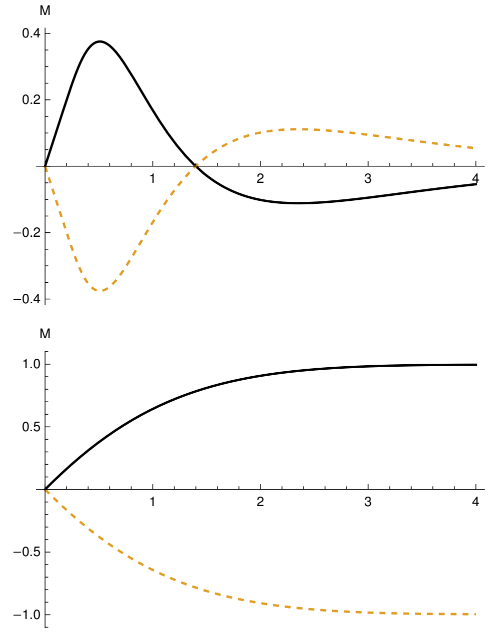

FIG. 21. The mass profiles for potentials in Eq. (50) and Eq. (56) under signature symmetry (+m solid,-m dashed).

since the base metric also changes. So Eq. (40) becomes

$$\begin{aligned} g_a &= \left(e^{\zeta}\right)^{\intercal} \cdot \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \mathrm{Sin}^2(\theta) \end{pmatrix} \cdot e^{\zeta} \\ &= \begin{pmatrix} -e^{2\Phi} & 0 & 0 & 0 \\ 0 & e^{-2\Phi} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \mathrm{Sin}^2(\theta) \end{pmatrix}. \end{aligned} \tag{133}$$

As shown in Fig. (21) the mass profiles for potentials Eq. (50) and Eq. (56) have perfect symmetry under signature reflection. Since antihydrogen has been considered a CPT conjugate of hydrogen, symmetry now requires CPTg as the new conjugacy.

V. SPECULATION

A. Big Bang

Big Bang

Matter must have separated from antimatter in the early epochs of the universe. Assume the universe started out symmetric. Consider matter and antimatter evenly dispersed. As an example consider a simple cubic lattice like salt with matter at the sodium sites and antimatter at the chloride sites. Such a configuration would have zero energy. It would also be highly unstable. If the lattice spacing was very small the metric would be zero, which challenges the notion of lattice spacing. A vanishingly small perturbation would start the separation with matter and antimatter segregating as space and time come into being. An elementary simulation of this can be found at https://thematterofspace.com/. Each specie begins to implode to a high density as they continue to separate. The implosion imparts kinetic energy to each specie which then causes an expansion. Of course a detailed cosmology needs to be built on this, but something like this must have happened. It also has some advantages over current cosmologies. The prevalent picture is that all matter comes into existence instantly at an infinitely high temperature singularity. Once again singularities and infinities are unphysical. This model has the initial condition of nothingness. Although there is initially no space or time, it might be said that it all started with an infinitesimally small perturbation an infinitely long time ago, just as a manner of speaking.

The mathematical model of this is obtained from Eq. (52). However, at t=0 no perturbation of the metric is small so Eq. (14) would now be

$$d\bar{x}^{\mu} = \boldsymbol{\zeta}^{\mu}_{\nu} dx^{\nu} \tag{134}$$

making Eq. (16)

$$g_{\mu\nu} = \boldsymbol{\zeta}^{\mu}_{\lambda} \bar{g}_{\mu\nu} \boldsymbol{\zeta}^{\nu}_{\tau}. \tag{135}$$

So using the solution Eq. (52) with α=-$\pi/2$ in Eq. (121)

$$\Phi(r) = -m\mathrm{Sin}[\omega t] \frac{\left(e^{-k^{-}r} - e^{-k^{+}r}\right)}{r}. \tag{136}$$

The metric is zero on the boundary between the matter and antimatter "universes". This means there is no spacetime separation between them. If the Cosmological Principle holds, each event being equivalent, this implies a certain topology. The antimatter part would be beyond the horizon for all events.

B. Baryon-Lepton Symmetry

Methodology Summary

Everything herein follows solely from the three axioms.

Ontology

A.1 is really all that is needed. ***A***.2 should be a requirement of any theory. ***A***.2 was needed to help deduce the implementation of ***A***.1. ***A***.1 is an enormous ontological simplification of physics. What can be said to exist is spacetime and its distortions.

Epistemology

This entire "theory" is mainly just the hypothesis that spacetime is never empty and therefore the EFE is an identity rather than an equation:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} \equiv \frac{8\pi\kappa}{c^2} T_{\mu\nu}, \tag{137}$$

which completely changes the epistemology of GR. Historically T has been taken to contain all sources of energy - except gravitational; gravity was "accounted for" on the left side of the equation. This has not been satisfactory from either a unification or a quantization point of view. Now all the forces are on the left as gauge fields, treated in the same way. This results in the observable, T, as all the energy that is the source of the gravitational field.

As mentioned above, the metric tensor must be known in order to calculate the correct relativistic value for T. So to solve for the metric tensor in this circular conundrum, both the metric and the matter density must be determined together along with symmetry conditions and an equation of state. An equation of state is provided apart from the field equation and therefore apart from the general theory of relativity, even if it is expressed in covariant form. The Correspondence Principle historically has been used by postulating that the limit of the left hand side of the EFE is equal to the classical limit of the right hand side in the weak-slow approximation. In this way

$$G_{\mu\nu} = \frac{8\pi\kappa}{c^2} T_{\mu\nu} \longrightarrow \nabla^2\Phi = 4\pi\rho, \tag{138}$$

which was initially obtained by reverse-engineering the Poisson equation.

This methodology uses the historical approach in reverse. The EFE is now a wrapper for the base space fields. That encapsulation is what allows unification without modification of the mathematical structure of GR.

$$\begin{aligned} &\nabla^2\Phi = 4\pi\rho, F^{\mu\nu}_{;\nu} = j^{\mu}, \text{etc.}, \longrightarrow \\ &G_{\mu\nu}\left(g_{\alpha\beta}\left(\zeta^{\sigma}_{\ \tau}\right)\right) + \Lambda g_{\mu\nu}\left(\zeta^{\sigma}_{\ \tau}\right) \equiv \frac{8\pi\kappa}{c^2} T_{\mu\nu}. \end{aligned} \tag{139}$$

Complex quantum fields need to be represented as real functions of the coordinates to make the mapping. For example, a complex scalar field, Ψ can be expressed as

$$\bar{\Psi} = e^{-i\Lambda}\Psi \rightarrow \overline{\begin{pmatrix} \phi \\ \chi \end{pmatrix}} = \begin{pmatrix} \mathrm{Cos}[\Lambda] & \mathrm{Sin}[\Lambda] \\ -\mathrm{Sin}[\Lambda] & \mathrm{Cos}[\Lambda] \end{pmatrix} \begin{pmatrix} \phi \\ \chi \end{pmatrix} \tag{140}$$

for a gauge transformation using coupled real fields or

$$\Psi = \phi + i\chi = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \phi + \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \chi = \begin{pmatrix} \phi & \chi \\ -\chi & \phi \end{pmatrix}, \tag{141}$$

using matrix representations of 1 and i. In this case the result is the sum of a symmetric field representing the mass and an antisymmetric field representing the charge as expected for a complex scalar field. This can be extended to spinor fields, etc. There is much more work to be done to turn this into a complete theory.

Appendix: Geometrized Units

The electric field components of the EM tensor in SI units is E/c. This has units of kg/(s C). So this is the unit of the proportionality constant, η, in Eq. (17) between the EM field and its dimensionless displacement field. Table 1 contains a sample of physical quantities in these new MSI units.

TABLE I. A sample of physical quantities in these new MSI units.

Quantity	Symbol	Unit
Electric field	E	m/s
Magnetic field	B	1
Vector potential	A	m
Charge	q	kg/s
Permittivity	ϵ	kg/m^3
Permeability	μ	m^2/N
Magnetic flux	Φ	m^2
Momentum	q A	$kg\,m/s$

This provides a view of spacetime such that the speed of light is

$$c = \sqrt{\frac{1\,/\mu_0}{\epsilon_0}} \sim \sqrt{\frac{B}{\rho}}, \tag{A.1}$$

where B is the bulk modulus and ρ the density as is typical for materials. So ϵ_0 has the role of density and μ_0 has units of compressibility in this picture of spacetime.

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